1997

Auction implementations using LaGrangian Relaxation, Interior-Point Linear Programming, and Upper-Bound Linear Programming

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Auction implementations using LaGrangian Relaxation, Interior-Point Linear Programming, and Upper-Bound Linear Programming

by

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A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

Major: Electrical Engineering

Major Professor: Gerald B. Sheblé

Iowa State University
Ames, Iowa

1997

Graduate College
Iowa State University

This is to certify that the Master's thesis of

Somgiat Dekrajangpetch

has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy

DEDICATION

To my parents,

Somkuan Dekrajangpetch and Pornpimol Sae-Heung.

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GLOSSARY OF TERMS

ANCILCO	Ancillary services company
BROCO	Broker company
DISTCO	Distribution company
EDC	Economic dispatch calculation
EMA	Energy management agent
ENSERVCO	Energy services company
GENCO	Generation company
ICA	Independent contract administrator
IPLP	Interior-point linear programming
IP	Interior-point programming
IPP	Independent power producer
ISO	Independent system operator
KKT	Karush-Kuhn-Tucker conditions
LP	Linear programming
LR	LaGrangian relaxation
NERC	North American Electric Reliability Council
POOLCO	Pool company
pu	per unit
TRANSCO	Transmission company
UBLP	Upper-Bound linear programming
UC	Unit commitment

ACKNOWLEDGEMENTS

I would like to thank my major professor, Dr. Gerald B. Sheblé, for providing valuable guidance, advice, and encouragement throughout my thesis. I also thank him for providing financial support for my study at Iowa State University. I would like to express my gratitude to Charles W. Richter for his time and effort in reviewing this thesis. I am grateful to Hudsadin Eamsherangkul for his help and support. Finally, I would like to thank my colleagues, Jayant Kumar, Charles W. Richter, Kah-Hoe Ng, and Mona Bisat for their useful discussions.

1 INTRODUCTION

In the U.S. electric power has been supplied by vertically integrated monopolistic utilities for a long time. A cost-based approach has been used for electrical pricing. To achieve economic efficiency of production and operation, inter-utility power interchange is prevalent between the utilities. The utilities having more expensive production buy energy from the utilities with less expensive production that have excessive capacity through interconnections. Energy brokerage systems are a well-known method used for power interchange. Under the traditional brokerage system, power interchange transactions are set up by central brokers in each period, e.g., hourly. The central brokers match the bids subject to certain rules and announce the accepted transactions. The other prevalent form of power interchange can be seen in power pools. Power pools are coordinated groups of utilities in which centralized unit commitment (UC) is performed across the entire power pool to have greatest savings.

Presently the electric power industry in the U.S. is restructuring to be more competitive. The cost-based approach to developing electricity rates will be changed to be price-based and auctions are considered to be a promising pricing mechanism for the competitive market. Various types of auctions have been proposed for use in the electric power market. This thesis focuses on certain types of auctions which will be described in this chapter.

1.1 Contents of this thesis

The purpose of this thesis is to show how to implement single-sided and double-sided auctions using various techniques and to describe the problems associated with using LaGrangian Relaxation to implement auctions. An algorithm is also developed for Interior-Point Linear Programming (IPLP) such that IPLP can find the exact optimal solution and sensitivity analysis can be performed with IPLP. The techniques considered in this thesis are LaGrangian Relaxation (LR), Interior-Point Linear Programming (IPLP), and Upper-Bound Linear Programming (UBLP). To implement auctions with these three techniques for this thesis, three computer programs are developed. These three computer programs are written in MATLAB.The details of the three computer programs will be described in this thesis.

The thesis is arranged as follows: the remainder of chapter 1 gives an overview of electric power market framework and the four types of auctions considered in this thesis. Chapter 2 reviews the previous work which has been done in areas related to this thesis. Chapter 3 explains the three techniques used in implementing auctions in this thesis. For each technique, the basic concepts, the algorithms, and the formulations applying to certain types of auctions are described. The problems associated with implementing type 1 and 2 auctions using LR are described in the LR section. Chapter 4 shows the results of illustrative auction examples tested on a six-bus system. The test cases showing the auction implementation problems associated with the use of LR are also included in chapter 4. Chapter 5 presents conclusions of this thesis. The appendix shows the six-bus system and the system data used to yield the results in chapter 4.

1.2 Framework

In this thesis, the framework of electric power market is based on Sheblé et al. [1]. The framework is shown in Figure 1.1. There are three main participants, generation companies (GENCOs), transmission companies(TRANSCOs), and distribution companies (DISTCOs). GENCOs sell energy to DISTCOs through transmission lines owned by TRANSCOs. The independent contract administrator (ICA) matches bids subject to the standards set up by NERC to maintain the systems in an efficient, secure, and reliable status. The participants can submit bids to the ICA directly or to energy management agents (EMAs) or broker companies (BROCOs). EMAs and BROCOs are power marketers. They are very similar to each other except that BROCOs deal more with bilateral contracts. EMAs or BROCOs broker transactions between the buyers and sellers that can agree on contracts and send bids from the remaining participants to the ICA to participate in the auction. Ancillary service companies (ANCILCOs) provide ancillary services for security and reliability of transmission systems. Energy Services Companies (ENSERVCOs) provide the services for ensuring high quality and reliable energy for customers to buy. ANCILCOs provide services for transmission systems while ENSERVCOs provide services for distribution systems. Note that for the test cases included in this thesis, the interactions of the TRANSCOs are ignored, but could easily be included.

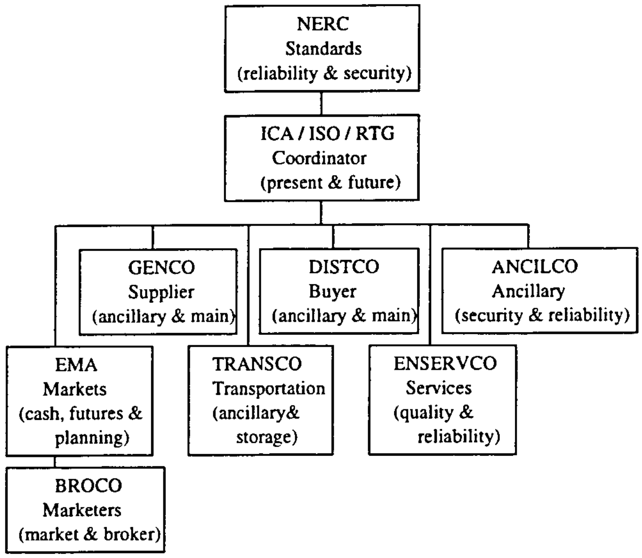

Figure 1.1 Electric power market framework

1.3 Auctions

This thesis considers two main types of auctions, singled-sided and double-sided auctions. Each type of auction is analyzed under two scenarios. Each scenario is based on what GENCOs and IPPs (and DISTCOs) submit to ICA, bids or fuel cost curves in case of GENCOs or IPPs, and bids or revenue curves in case of DISTCOs. Thus, there are four auction scenarios considered in this thesis. A generic diagram of all four scenarios is shown in Figure 1.2. For this thesis, the results and discussion are based on the diagram in Figure 1.2. The differences between each type of auction is shown in Table 1.1.

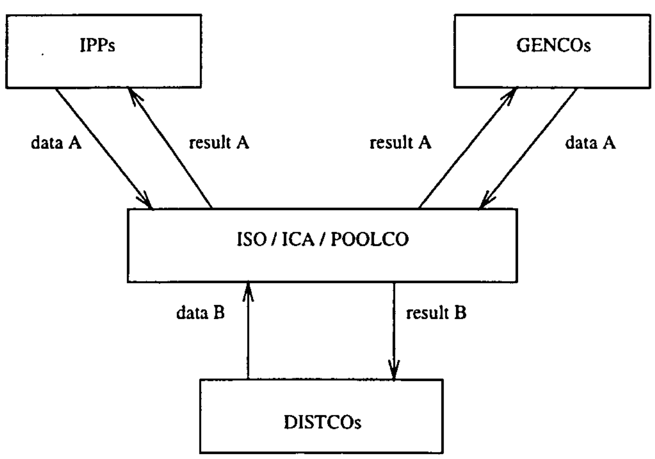

Figure 1.2 Auction diagram

Table 1.1 Four types of auctions considered in this thesis

Type	Description	data A	data B	result A	result B
Type 1	Single-Sided, Cost-Based	Cost Models	Loads/hour	Schedules	Costs
Type 2	Double-Sided, Cost-Based	Cost Models	Revenue Models	Schedules	Costs
Type 3	Single-Sided, Price-Based	Submitted Bids	Loads/hour	Accepted Bids	Costs
Type 4	Double-Sided, Price-Based	Submitted Bids	Submitted Bids	Accepted Bids	Accepted Bids

In McAfee et al. [2], an auction is defined as "a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants". From this definition, type 1 and 2 auctions as described in the thesis can be considered as auctions in the sense that the electricity allocation and prices are determined on the basis of bids from IPPs, GENCOs, (and DISTCOs). The point of contention is that in the old environment the bids of IPPs and GENCOs are mandated to be the true cost functions (plus some rate of return) and the bids of DISTCOs are mandated to be the true revenue functions (plus some rate of return). The participants were still bidding, but they were restricted to bidding their costs. It is mainly a matter of semantics, and for the purposes of discussion and implementation this thesis assumes type 1 and 2 auctions to really be auctions.

Comparing the inter-utility power interchange procedures in the old environment, power pools and energy brokerage systems, to type 1, 2, 3, and 4 auctions, power pools are equivalent to type 1 auctions. Energy brokerage systems are similar to type 3 and 4 auctions except that the prices bid in the energy brokerage systems have to be based on the true cost curves plus some rate of return while the prices bid in type 3 and 4 auctions can be any value the bidders desire.

Throughout this thesis, type 1, 2, 3, and 4 auctions will be used to refer to the various cases described in Table 1.1. Type 1 and 3 auctions are single-sided auctions and type 2 and 4 auctions are double-sided auctions. For type 1 and 3 auctions, DISTCOs do not have a chance to bid. They only submit the hourly load and get the energy at the same price as other DISTCOs. For type 2 auctions, DISTCOs submit their revenue models. The concept of revenue models will be discussed in Chapter 3.

2 LITERATURE REVIEW

For the new competitive electric power market, auctions have been proposed and are considered to be a promising method for pricing. Sheblé [3] outlined a method to use auction systems in cash and future markets to provide reserve margins for generator and transmission line forced outages. Wollenberg et al. [4] developed a document which discussed interesting technical issues for electric power market in a response to the Federal Energy Regulatory Commission 's Notice of Proposed Regulation of March 29, 1995. Post [5] gave a complete explanation of various types of auctions. In this thesis, three methods, LR, IPLP, and UBLP, are used to implement auctions. Previous research that has been done in related areas is described below.

2.1 LaGrangian Relaxation (LR)

Fisher [6] described the basic formulation and discussed interesting issues of applying LR to solve integer programming problems. Three methods for updating LaGrange multipliers, the subgradient method, column generation techniques of the simplex method, and multiplier adjustment methods, were also reviewed in the paper. There are many methods for updating LaGrange multipliers. Among these methods, the subgradient method is promising and is widely used. Fisher [7] presented a generic algorithm of using LR together with the branch and bound method to solve integer programming problems. The general form and numerical example were also shown in the paper.

LR was applied to implement the UC problem which is a large-scale mixed integer programming problem. Sheblé et al. [8] presented an overview of the literature in the field of UC. Merlin et al. [9] introduced an algorithm to completely solve large scale UC problems without incorporating the branch and bound method. Bard [10] mentioned that including ramping constraints in UC increased the computational burden dramatically in constructing good feasible solutions, based on the proposed algorithm. Zhuang et al. [11] presented an algorithm comprising three phases. In the first phase, the LaGrangian dual of the UC problem was solved by sufficiently many subgradient iterations. In the second phase, a systematic procedure was developed to search for suboptimal reserve-feasible dual solutions by intelligently adjusting the LaGrange multipliers. In the third phase, the economic dispatch calculation (EDC) was performed with the schedule from the second phase. Aoki et al. [12] implemented optimal long-term UC in large scale systems with LR. The long-term UC was implemented because fuel constrained thermal and pumped-storage hydro units were included. Demand, reserve, and fuel constraints, were relaxed. The variable matrix method was used instead of the subgradient method to achieve better convergence in this long-term UC. Virmani et al. [13] observed some implementation aspects of LR while applying it to realistic and practical UC problems and also discussed handling identical generation units, by committing them as a group or adjusting their heat rates slightly to make them distinct and then committing them separately. Lee et al. [14] presented a method for solving reserve constrained EDC when some on-line generators had prohibited operating zones, which made the decision space non-convex. The method decomposed the prohibited operating zones into a small number of subsets such that each of the associated EDC problems was either infeasible or solvable by the conventional LR method. When comparing all the feasible costs of the feasible subproblems, the optimal solution is the least cost one. Ferreira [15] derived a bound on the duality gap for thermal UC under the assumption of no minimum commitment times. Yan et al. [16] scheduled hydrothermal power systems by relaxing system-wide demand and reserve constraints and then decomposing the problem into hydro and thermal unit subproblems. Comparison were made that this new coordinated hydro and thermal unit scheduling generated lower total costs and required less computation time than previous done by Yan et al. where thermal units were scheduled by using LR and hydro units by heuristics. Jeloka [17] implemented UC using LR. Slawaji et al. [18] proposed an approach to solve UC of thermal units for large scale power system. The approach classified the units having identical input-output characteristics into the same group and represented each group by one unit. Guan et al. [19] focused on the solution methodology for pumped-storage units by relaxing pond level constraints in scheduling. Wang et al. [20] presented the application of a mathematical method to generator scheduling, in which ramp rate limits were added in constraints of UC and the cost of fatigue effect was included in the objective function of EDC. The UC was solved with LR and the EDC was solved with LP. El-Keib et al. [21] proposed an algorithm to solve the environmentally constrained EDC problem. The algorithm can handle a large number of various types of linear and nonlinear environmental constraints. Svoboda et al. [22] showed how to incorporate the endogenously priced resources into short-term scheduling. This was different from the prevailing method in which the endogenously priced resources were activated through post-dispatch price signals derived from the scheduling. The simulation study showed that integrated scheduling could produce significant improvements in operations and costs. Wang et al. [23] included ramping costs in the objective function of optimal generation scheduling. Baldick [24] formulated and proposed an algorithm for generalized UC. This generalized UC was able to handle many types of constraints. Guan et al. [25] approximated linear cost functions of subproblems in hydrothermal scheduling problems as non-linear functions so that the solution of subproblems did not oscillate. Peterson et al. [26] extended the LR algorithm to account for crew constraints in UC. Prasannan et al. [27] included the seller's revenue in the objective function of UC to integrate decision in offering transactions with system scheduling. The seller's nonlinear revenues were used to approximate their linear revenue functions to solve the problem efficiently. Lin et al. [28] included purchase cost in the objective function to integrate decisions on offering purchase transactions with system scheduling. Gjengedal [29] incorporated emission constraints in the UC to achieve daily or weekly emission targets. Bos et al. [30] developed an algorithm for combining electricity and heat UC. Heat storage devices were incorporated in the UC and the subproblems of the heat storage devices were converted into LP problems. Ruzic et al. [31, 32] presented a new flexible approach for short-term hydro-thermal coordination in UC problems. The paper presented two case-studies having completely different thermal hydro systems to show flexibility of the proposed approach.

The above references ([9] to [32]) developed algorithms and LaGrange multiplier updating procedures to be appropriate with different objective functions and constraints. The basic formulations, algorithms, and LaGrange multiplier updating procedures can also be seen in Sheblé [33] and Wood et al. [34].

2.2 Interior-Point Linear Programming (IPLP)

Arbel [35] gave explanations, formulations, and algorithms of the primal, dual, and primal-dual affine-scaling interior-point linear programming method. Hertog [36] explained the logarithmic barrier method, the center method. Interior-Point Programming (IP) method was applied to power system problems. Some power system problems were solved by interior point linear programming [37] while some were solved by interior point quadratic programming [38, 39, 40, 41, 42, 43]. Wu et al. [38], Momoh et al. [41], and Torres et al. [43] used IP to solve OPF problems. Yan et al. [37] applied IP to solve security-constrained EDC problem. Granville [39] handled optimal reactive dispatch problem by IPP. Wei et al. [42] utilized IP to solve OPF and EDC problems. Momoh et al. [40] implemented EDC and optimal VAR dispatch by IP.

IPLP is not able to find an exact optimal solution, but rather finds a solution that is very close to the optimal solution. Another drawback is that sensitivity analysis cannot be done with IPLP. Marsten et al. [44] used a special simplex algorithm using the concept of super-basic variables to recover the optimal basis after terminating from applying dual affine interior point algorithm to solve optimization problems. The drawback of the basic recovery method described in [44] was that it was computationally expensive. In experiments Marsten et al. [44] pointed out that recovery of an optimal basis could require more execution time than was required to solve the problem by dual affine method.

2.3 Linear Programming: Simplex method

This subsection summarizes the previous research that solves related power system problems using the simplex LP method. This summary covers the generalized simplex method and is not limited only to UBLP. Bazaraa et al. [45], Hillier et al. [46], and Luenberger [47] explained the basic concepts, applications, and implementation issues. Fahd et al. [48] implemented an energy brokerage system using LP. Fahd et al. [49] presented an interchange brokerage system based on OPF solution by using LP. Smith [50] developed a model for real-time pricing of electric power using LP. Roy [51] used goal-programming to determine optimal pricing for inter-area energy exchange. Post et al. [52] used LP to implement one-sided auctions, bidding by buyers, with sellers' reservation prices, without simultaneous consideration of network constraints. Chattopadhyay [53] presented a LP formulation for energy brokerage system with emission trading and allocation of cost savings. Kumar et al. [54] proposed a framework for an energy brokerage system with reserve margin and transmission losses using LP. The network constraints were simultaneously considered in solving auctions. Kumar [55] developed a framework for market-based pricing of ancillary services in electric power transactions. Kumar used LP to yield the illustrative examples contained in [55].

The above references ([48] to [55]) used LP to implement auction, brokerage, and pricing problems. Apart from these problems, the following related problems were also solved by LP. LP was used to solve OPF problems [56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66], rescheduling problems [67, 68], and reactive power control problems [69, 70, 71, 72, 73]. Bosch [74] used LP to implement EDC. Zhang et al. [75] and Huang et al. [76] applied LP to solve security constrained EDC problems. El-Keib et al. [77] handled environmentally constrained EDC using LP.

3 THREE METHODS FOR IMPLEMENTING AUCTIONS

Before explaining each of the three methods used to implement auctions in this thesis, the common notations are described here. The bold type letter indicates a vector or a matrix. The normal letter indicates scalar. $\mathbf{M}^T$ signifies the transpose of matrix $\mathbf{M}$. $\mathbf{M}^{-1}$ signifies the inverse of matrix $\mathbf{M}$. $\mathbf{M}_{ij}$ is the element of matrix $\mathbf{M}$ at the ith row and jth column. $\mathbf{I_m}$ is a m x m identity matrix. The Euclidean norm of vector $\mathbf{v}$ is represented by $norm(\mathbf{v})$. $\mathbf{v}_i$ is the ith component of vector $\mathbf{v}$. The magnitude of vector $\mathbf{v}$ is denoted by $|\mathbf{v}|$. $|\alpha|$ is the absolute of scalar α.

3.1 LaGrangian Relaxation (LR)

LaGrangian Relaxation (LR) is an optimization technique which decomposes the main complex mathematical programming into simple subproblems that are additively separable by relaxing the hard constraints, e.g. coupled constraints. Each subproblem is coupled through common LaGrange multipliers. Each subproblem is solved separately, and the complete problem is solved by updating the LaGrange multipliers at each iteration until a near-optimal solution is found. LR has been successfully applied to various problems. For electric power, LR has been applied to the unit commitment (UC) problem. The LR algorithm is successful since a LaGrange multiplier updating procedure has been suitably developed to converge efficiently. Many methods have been developed for updating the LaGrange multipliers. Among these methods, the subgradient method is promising and is widely used in UC. For the purpose of this thesis, the subgradient method is considered good enough for updating LaGrange multipliers.

LR has many advantages over other methods. For example, in UC, the computational requirement of using LR varies linearly with number of generation units, N and stages, T while the computational requirement of dynamic programming (DP) varies exponentially with N and T, $(2^N - 1)^T$. It is also easy to handle additional constraints in LR if they are additively separable to the problem. Another set of LaGrange multipliers is required for relaxing a set of additional constraints. However, LR has some weaknesses when it comes to convergence. The solution found by LR might not be feasible nor near-optimal if LaGrange multipliers have not been updated properly.

LR is used to implement type 1, 2, 3, and 4 auctions in this thesis. The formulations, algorithms, and LaGrange multiplier updating procedures of LR for application to different types of auctions are shown below. The implementation problems associated with implementing type 1 and 2 auctions using LR are described at the last part of this section.

3.1.1 Applying LR to type 1 auctions

In type 1 auctions, all GENCOs and IPPs submit their generating cost models to the ICA, and DISTCOs submit their hourly loads to the ICA. Then the ICA performs the auction by UC analysis using LR for the system in the specified period, 24 or 168 hours. In other words, the ICA performs auctions with the LR based UC procedure because the formulation of the LR-based type 1 auction is the same as that of the LR-based UC. After the ICA finds the optimal solution, the optimal schedule is given to each GENCO and IPP, and the optimal cost is given to each DISTCO.

This thesis uses the formulation, algorithm, and LaGrange multiplier updating procedure of LR-based auctions which are modified from those used for LR-based UC as described in Merlin et al. [9]. For simplicity, the following assumptions are being made. The spinning reserve constraints have been neglected. The fuel cost is assumed to be a quadratic function. The start-up cost is assumed to be constant for all units which also represents the transition cost because the shut-down cost is assumed to be zero. These assumptions of the spinning reserve constraints, the fuel cost, and the start-up cost will be used throughout this thesis.

Before explaining the formulation, notations of the common symbols used in this subsection are described below. These common symbols will also be used in subsection 3.1.2.

T	number of total stages
$stup_i^t$	start-up cost of unit i from stage $t-1$ to t (\$/h)
N_g	number of GENCOs
N_d	number of DISTCOs
P_{ig}^t	power sold by GENCO i at stage t (MW)
P_{id}^t	power bought by DISTCO i at stage t (MW)
P_{ig}^{min}	minimum capacity of GENCO i (MW)
P_{ig}^{max}	maximum capacity of GENCO i (MW)
L_i^{min}	minimum load capacity of DISTCO i (MW)
L_i^{max}	maximum load capacity of DISTCO i (MW)

$F_i(P_{ig}^t)$	fuel cost for GENCO i, assumed quadratic function (\$/h)
	$F_i(P_{ig}^t) = a_{ig}{P_{ig}^t}^2 + b_{ig}P_{ig}^t + c_{ig}$
	a_{ig}, b_{ig}, and c_{ig} are non-negative coefficients.
$R_i(P_{id}^t)$	revenue for DISTCO i, assumed quadratic function (\$/h)
	$R_i(P_{id}^t) = -a_{id}{P_{id}^t}^2 + b_{id}P_{id}^t + c_{id}$
	a_{id}, b_{id}, and c_{id} are non-negative coefficients.
$load^t$	demand at time t (MW)
λ^t	LaGrange multiplier at time t (\$/MWh)
λ	vector containing λ^t from $t = 1$ to $t = T$ (\$/MWh)
u_{ig}^t, u_{id}^t	index showing status of GENCO and DISTCO i at stage t: 1=on(selected), and 0=off(not selected)
$pobj$	value of primal objective function (\$)
$dobj$	value of dual objective function (\$)

3.1.1.1 Formulation

For ease of notation, the start-up cost of unit i from stage $t-1$ to t is simply shown by $stup_i^t$. The $stup_i^t$ will exist only when $u_{ig}^{t-1} = 0$ and $u_{ig}^t = 1$. Formulation of the auction is shown below:

Primal problem

$$\min_{u_{ig}^t, P_{ig}^t} pobj(u_{ig}^t, P_{ig}^t) \tag{3.1}$$

where

$$pobj(u_{ig}^t, P_{ig}^t) = \sum_{t=1}^{T} \sum_{i=1}^{N_g} [F_i(P_{ig}^t)u_{ig}^t + stup_i^t] \tag{3.2}$$

subject to

power balance constraints

$$\sum_{i=1}^{N_g} P_{ig}^t u_{ig}^t = load^t \qquad t = 1, 2, ..., T \tag{3.3}$$

unit capacity constraints

$$P_{ig}^{min} \leq P_{ig}^t \leq P_{ig}^{max} \qquad i = 1, 2, ..., N_g \qquad t = 1, 2, ..., T \tag{3.4}$$

minimum up and down time constraints (3.5)

unit status constraints

$$u_{ig}^t = 0, 1 \qquad i = 1, 2, ..., N_g \qquad t = 1, 2, ..., T \tag{3.6}$$

Derivation from primal to dual problems

For the primal problem, the power balance constraints, (3.3), may be relaxed and the LaGrange function can be written as:

$$\begin{aligned} L(u_{ig}^t, P_{ig}^t, \lambda^t) &= \sum_{t=1}^{T}\sum_{i=1}^{N_g}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t] + \sum_{t=1}^{T}\lambda^t(load^t - \sum_{i=1}^{N_g}P_{ig}^t u_{ig}^t) \quad (3.7)\\ &= \sum_{t=1}^{T}\lambda^t load^t + \sum_{t=1}^{T}\sum_{i=1}^{N_g}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t] \quad (3.8)\\ &= \sum_{t=1}^{T}\lambda^t load^t + \sum_{i=1}^{N_g}\sum_{t=1}^{T}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t] \quad (3.9) \end{aligned}$$

The dual objective function wants to

$$\max_{\lambda^t}[\min_{u_{ig}^t, P_{ig}^t} L(u_{ig}^t, P_{ig}^t, \lambda^t)] \quad (3.10)$$

From (3.9), for each set of λ^t, $\sum_{t=1}^{T}\lambda^t load^t$ is a constant term ;thus,

$$\begin{aligned} \min_{u_{ig}^t, P_{ig}^t} L(u_{ig}^t, P_{ig}^t, \lambda^t) &= \sum_{t=1}^{T}\lambda^t load^t + \min_{u_{ig}^t, P_{ig}^t}(\sum_{i=1}^{N_g}\sum_{t=1}^{T}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t]) \quad (3.11)\\ &= \sum_{t=1}^{T}\lambda^t load^t + \sum_{i=1}^{N_g}\min_{u_{ig}^t, P_{ig}^t}(\sum_{t=1}^{T}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t]) \quad (3.12) \end{aligned}$$

From this manipulation, the dual problem can be shown as follows:

Dual problem

$$\max_{\lambda^t} dobj(\lambda^t) \quad (3.13)$$

where

$$dobj(\lambda^t) = \sum_{t=1}^{T}\lambda^t load^t + d(\lambda^t) \quad (3.14)$$

and

$$d(\lambda^t) = \sum_{i=1}^{N_g}\min_{u_{ig}^t, P_{ig}^t}(\sum_{t=1}^{T}[F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t]) \quad (3.15)$$

where (3.15) is minimized, subject to (3.4), (3.5), and (3.6). Once (3.15) is achieved, the complex minimization problem can be simply solved by minimization for each unit separately, which can be solved by two state DP. The minimization for each GENCO is done through (3.16) subject to (3.4), (3.5), and (3.6) and can be shown below:

$$\min_{u_{ig}^t, P_{ig}^t} [\sum_{t=1}^{T} F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t u_{ig}^t] \tag{3.16}$$

3.1.1.2 Algorithm and computer program

The algorithm used for LR in this thesis is shown in Fig. 3.1. There are two criteria for terminating the algorithm. First the algorithm will terminate when duality gap is less than or equal to 0.026 ($\epsilon =$ 0.026). Second the algorithm will terminate when number of iterations exceeds 100 ($iter^{max} = 100$). The reason that a rather large number, 100 is used for the small studied system is that the cases studied in subsection 3.1.5 are those in which LR has difficulties in converging to the optimal solution.

A general computer program for LR-based type 1 auctions is developed based on the algorithm in Fig 3.1. The program is written in MATLAB. The program is flexible so that it can be modified for use with LR-based type 2 auctions .

3.1.1.3 Updating procedures

The subgradient technique is used for updating LaGrange multipliers. Each λ^t is updated according to (3.17).

$$\lambda^t = max[\lambda^t + \frac{pdif^t}{(\alpha + \beta * iter) * norm(pdif)}, 0] \tag{3.17}$$

α and β are constants and $pdif^t$ can be defined at (3.18),

$$pdif^t = load^t - \sum_{i=1}^{N_g} P_{ig}^t \tag{3.18}$$

and so $pdif$ is a vector containing $pdif^t$ from $t = 1$ to T. $norm(pdif)$ is the Euclidean norm of vector $pdif$. P_{ig}^t here is calculated from DP, not from EDC.

The values of α and β can be divided into two categories according to the sign of $pdif^t$ as follows:

Category 1: $pdif^t > 0$: α=0.02, β=0.05.

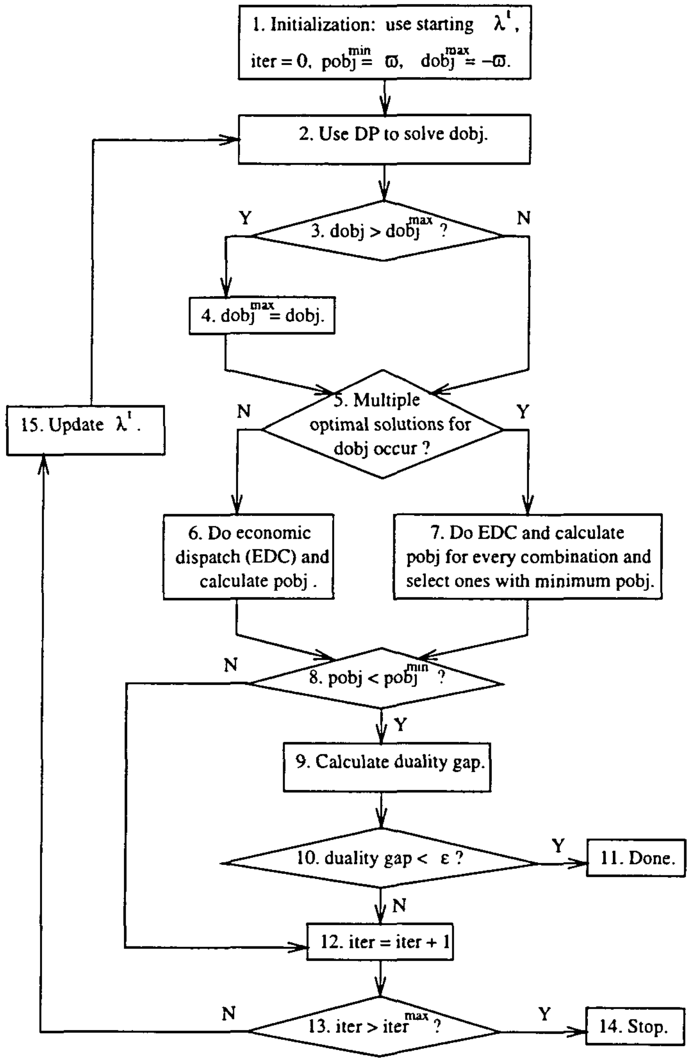

Figure 3.1 Algorithm for LR-based type 1 auctions

Category 2: $pdif^t \leq 0$: α=0.5, β=0.25.

α and β when $pdif^t \leq 0$ are rather large, and larger than those when $pdif^t > 0$, to make LR converge suitably for the cases studied in subsection 3.1.5.

3.1.2 Applying LR to type 2 auctions

In type 2 auctions, instead of submitting the hourly loads, DISTCOs submit revenue models to the ICA. The ICA implements the auctions so that the difference between the total revenue of DISTCOs and the total production cost of GENCOs is maximized. This subsection will describe the concept of revenue models first, followed by formulation, algorithm, and updating procedure of LR.

3.1.2.1 Concept of DISTCOs' revenue models

The revenue model of a DISTCO will be explained by the revenue curve. The revenue curve of a DISTCO is the curve describing the DISTCO's revenue and the amount of power sold by the DISTCO or load. The model of the revenue curve in this thesis is quadratic and concave which has downward sloping linear rate of revenue. The model has two basic characteristics. First, the revenue increases with increasing amount of power sold. Second, the rate of revenue with reference to the amount of power sold, the decremental revenue, decreases with increasing amount of power sold. In other words, the price of power is less expensive when customers buy greater amounts of power. General forms of mathematical functions of revenue and decremental revenue are shown in (3.19) and (3.20) respectively. The revenue function of a DISTCO changes with time because it depends on the demand and supply of the market. Fig. 3.2 and 3.3 show the curves of revenue and decremental revenue of a DISTCO having $R_i(P_{id}^t) = -0.0015 * {P_{id}^t}^2 + 12.05P_{id}^t + 450$. The revenue curve in Fig. 3.2 is actually quadratic and concave. It looks like straight line because the values of the revenue axis are much greater than the those of the load axis.

$$R_i(P_{id}^t) = -a_{id}{P_{id}^t}^2 + b_{id}P_{id}^t + c_{id}; \quad a_{id}, b_{id}, c_{id} \geq 0 \tag{3.19}$$

$$\frac{dR_i(P_{id}^t)}{dP_{id}^t} = -2a_{id}P_{id}^t + b_{id} \tag{3.20}$$

The graphical method is a method for doing conventional EDC. The procedure of the graphical method is to find the aggregate incremental cost curve of all committed generating units and then the optimal incremental cost can be found from this aggregate curve at the level of total required capacity, i.e., total load. From this optimal incremental cost, the generating power of each committed unit can be found. This concept can be adapted for use with the EDC of LR-based type 2 auctions, at steps 6 and 7 of the algorithm shown in Fig. 3.1 by reversing the power of DISTCOs to be negative when finding the aggregate incremental curve. The optimal incremental price can be found at the level of zero total power according to (3.23), that will be described in subsection 3.1.2.2. This concept is used in the algorithm that will be described in subsection 3.1.2.3. Actually this concept can also be interpreted as the optimal price which can be found from the intersection of aggregate GENCOs' incremental cost curve and aggregate DISTCOs' decremental revenue curve. An example is shown in Fig. 3.4, in which

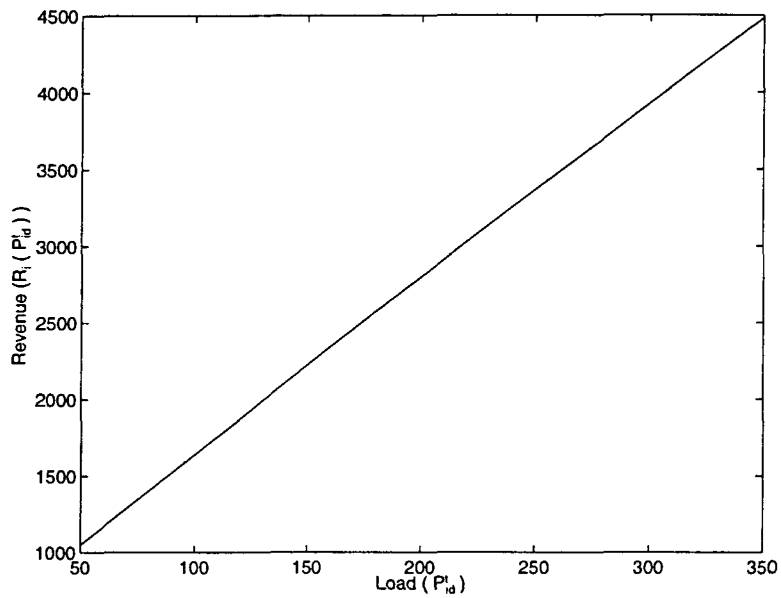

Figure 3.2 Revenue curve of a DISTCO

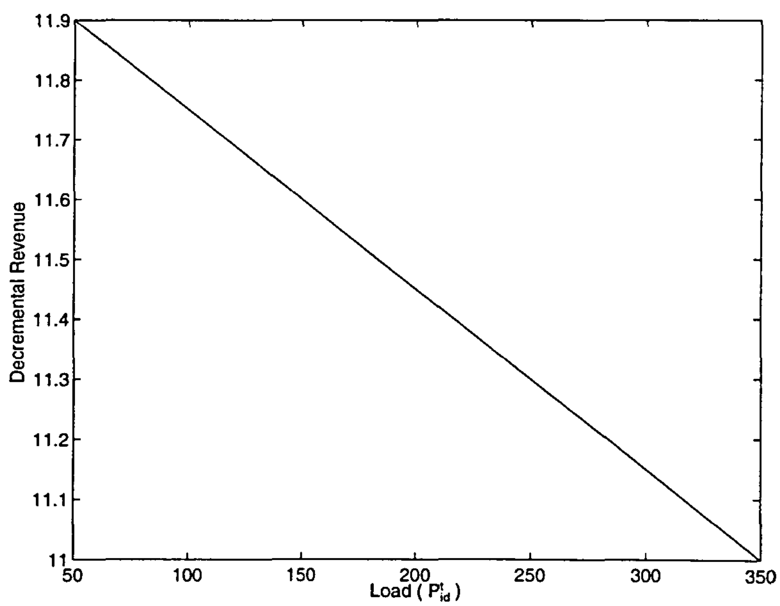

Figure 3.3 Decremental revenue of a DISTCO

the intersection of the aggregate incremental cost curve of GENCOs 1 and 2 and the aggregate decremental revenue curve of DISTCOs 1 and 2 are shown. The intersection shows that the optimal price is 9.6 and the optimal total power or load is 520 MW. The data of GENCOs 1, 2 and DISTCOs 1, 2 are shown in Table 3.1. These data come from the data of GENCOS and DISTCOs in stage 2 of section 4.1.2.

Table 3.1 Data of GENCOs and DISTCOs for illustrating the intersection of aggregate incremental cost and aggregate decremental revenue

Unit(i)	a_{ig}, a_{id}	b_{ig}, b_{id}	c_{ig}, c_{id}	P_{ig}^{min}, L_i^{min}	P_{ig}^{max}, L_i^{max}
GENCO 1	0.0025	8.00	300	100	400
GENCO 2	0.0050	6.00	100	50	200
DISTCO 1	-0.0015	12.05	450	50	350
DISTCO 2	-0.0035	10.79	300	50	200

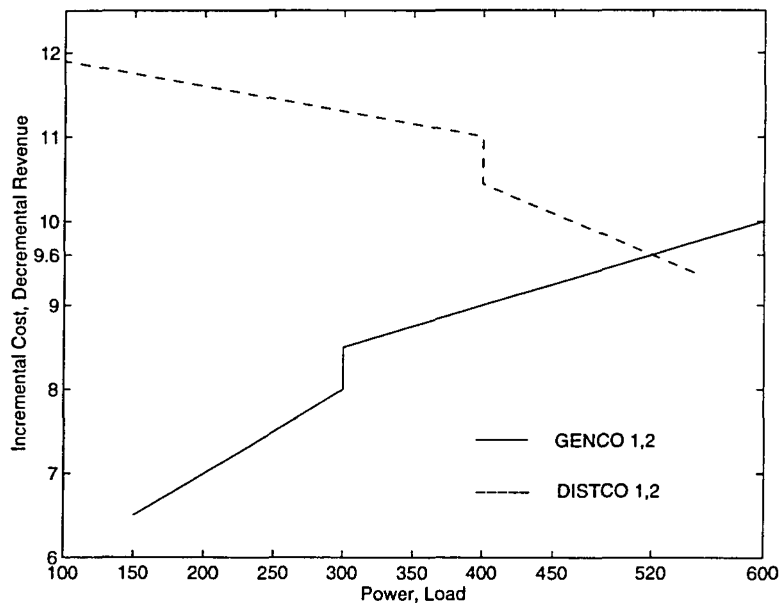

Figure 3.4 Intersection of aggregate incremental cost and aggregate decremental revenue

3.1.2.2 Formulation

The formulation below is developed for LR based type 2 auctions. DISTCOs do not have start-up costs.

Primal problem

$$\min_{u_{ig}^t, u_{id}^t, P_{ig}^t, P_{id}^t} pobj(u_{ig}^t, u_{id}^t, P_{ig}^t, P_{id}^t) \tag{3.21}$$

where

$$pobj = \sum_{t=1}^{T} \sum_{i=1}^{N_g} F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \sum_{t=1}^{T} \sum_{i=1}^{N_d} R_i(P_{id}^t)u_{id}^t \tag{3.22}$$

subject to

power balance constraints

$$\sum_{i=1}^{N_g} P_{ig}^t u_{ig}^t - \sum_{i=1}^{N_d} P_{id}^t u_{id}^t = 0 \qquad t = 1, 2, ..., T \tag{3.23}$$

unit capacity constraints

$$P_{ig}^{min} \leq P_{ig}^t \leq P_{ig}^{max} \qquad i = 1, 2, ..., N_g \quad t = 1, 2, ..., T \tag{3.24}$$

$$L_i^{min} \leq P_{id}^t \leq L_i^{max} \qquad i = 1, 2, ..., N_d \quad t = 1, 2, ..., T \tag{3.25}$$

minimum up and down time constraints (for only P_{ig}^t) (3.26)

unit status constraints

$$u_{ig}^t = 0, 1 \qquad i = 1, 2, ..., N_g \quad t = 1, 2, ..., T \tag{3.27}$$

$$u_{id}^t = 0, 1 \qquad i = 1, 2, ..., N_d \quad t = 1, 2, ..., T \tag{3.28}$$

For the primal problem the power balance constraints (3.23) may be relaxed and the LaGrange function, $L(u_{ig}^t, u_{id}^t, P_{ig}^t, P_{id}^t, \lambda^t)$ can be written as:

$$L = \sum_{t=1}^{T} \sum_{i=1}^{N_g} F_i(P_{ig}^t) u_{ig}^t + stup_i^t - \sum_{t=1}^{T} \sum_{i=1}^{N_d} R_i(P_{id}^t) u_{id}^t + \sum_{t=1}^{T} \lambda^t (\sum_{i=1}^{N_d} P_{id}^t u_{id}^t - \sum_{i=1}^{N_g} P_{ig}^t u_{ig}^t) \tag{3.29}$$

By rearranging, (3.29) can be written as:

$$L = \sum_{i=1}^{N_g} [\sum_{t=1}^{T} (F_i(P_{ig}^t) u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t)] + \sum_{i=1}^{N_d} [\sum_{t=1}^{T} (-R_i(P_{id}^t) u_{id}^t + \lambda^t P_{id}^t)] \tag{3.30}$$

From the derivation of the LaGrange function, the dual problem can be shown below.

Dual problem

$$\max_{\lambda^t} dobj(\lambda^t) \tag{3.31}$$

where

$$\begin{aligned} dobj(\lambda^t) \quad = \quad & \sum_{i=1}^{N_g} \min_{u_{ig}^t, P_{ig}^t} [\sum_{t=1}^{T} (F_i(P_{ig}^t) u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t)] \\ & + \sum_{i=1}^{N_d} \min_{u_{id}^t, P_{id}^t} [\sum_{t=1}^{T} (-R_i(P_{id}^t) u_{id}^t + \lambda^t P_{id}^t)] \end{aligned} \tag{3.32}$$

The objective function (3.32) is in the separable form where the complex minimization problem can be simply solved by minimizing for each unit separately. The minimization for each GENCO is accomplished by (3.33) subject to (3.24), (3.26), and (3.27). The minimization for each DISTCO is accomplished by (3.34) subject to (3.25) and (3.28). (3.33) and (3.34) can be solved by two state DP and they are shown below.

$$\min_{u_{ig}^t, P_{ig}^t} [\sum_{t=1}^{T}(F_i(P_{ig}^t)u_{ig}^t + stup_i^t - \lambda^t P_{ig}^t)] \tag{3.33}$$

$$\min_{u_{id}^t, P_{id}^t} [\sum_{t=1}^{T}(-R_i(P_{id}^t)u_{id}^t + \lambda^t P_{id}^t)] \tag{3.34}$$

3.1.2.3 Algorithm and computer program

The algorithm and the computer program used for LR-based type 2 auctions are modified slightly from those of LR-based type 1 auctions. Comparing the formulations of LR-based type 1 and 2 auctions, we see that they are very similar to each other except for three main differences. First, LR-based type 2 auctions maximize the difference between the total revenue of DISTCOs and the total production cost of GENCOs, which can be interpreted as minimizing the negative of the total revenue of DISTCOs and minimizing the total production cost of GENCOs. By multiplying the revenue of each DISTCO by -1, the objective function of type 2 auctions is a minimization function which can be implemented in the algorithm and the computer program of LR-based type 1 auctions. Second, a comparison of (3.23) with (3.3), reveals that (3.23) can be interpreted as summing all the P_{ig}^t and all the negative of P_{id}^t, -P_{id}^t, and equating $load^t$ to zero vector. The EDC in steps 6 and 7 of the algorithm also must be implemented in this way. Third, comparing the separable minimization for each GENCO of LR-based type 1 auctions, (3.16), with (3.33) the separable minimization for each GENCO of LR based type 2 auctions is the same. For DISTCOs in LR-based type 2 auctions , from (3.34), the separable minimization is very similar except for a sign reversal of the two terms of the separable objective function. Therefore, the algorithm and the computer program of LR-based type 1 auctions can be used with LR-based type 2 auctions as in the following procedure; reverse the sign of the revenue function of each DISTCO, reverse the signs of all Pid^t for (3.23), input $load^t$ as zero vector, and change from $-\lambda^t Pid^t$ to be $\lambda^t Pid^t$ for individual unit's minimization.

3.1.2.4 Updating procedure

The updating procedure of LR-based type 2 auctions is the same as that of LR-based type 1 auctions presented in subsection 3.1.1.3 except that the equation describing $pdif^t$ in (3.18) must be changed to

(3.35) and the values of α and β used for both cases, when $pdif^t > 0$ and when $pdif^t \leq 0$, are changed to 0.5 and 0.25 respectively.

$$pdif^t = \sum_{i=1}^{N_d} P_{id}^t - \sum_{i=1}^{N_g} P_{ig}^t \tag{3.35}$$

3.1.3 Applying LR to type 4 auctions

In type 4 auctions, GENCOs, IPPs, and DISTCOs submit their bids to the ICA. The ICA performs auctions by maximizing the surplus. The surplus is defined as the difference between the total revenue of DISTCOs and the total revenue of GENCOs and IPPs.

3.1.3.1 General type 4 auction formulation

The general formulation for type 4 auctions is based on the auction model for pricing reserve margins and transmission losses developed in Kumar's dissertation [55]. For type 3 auctions, the formulation is modified from that used for type 4 auctions. This is why the application of LR to type 4 auctions is described first in this subsection and then followed by the application of LR to type 3 auctions in subsection 3.1.4.

The common symbols for the general formulations of type 3 and 4 auctions will be described below. These symbols are common for all techniques applying to type 3 and 4 auctions.

To use (3.37), components of $\mathbf{\Delta P}$ and $\mathbf{\Delta \delta}$ must be very small; thus, all the quantities in the formulation are in per unit (pu), except that δ_i and $\mathbf{\Delta \delta}$ are in radian and c_{bj} and c_{si} are in \$/unit, e.g. \$/MWh.

c_{bj}	price of jth buyer's bid
c_{si}	price of ith seller's bid
ΔP_{bj}	accepted amount of power of jth buyer
ΔP_{si}	accepted amount of power of ith seller
n	number of buyers
m	number of sellers
$\mathbf{\Delta \delta}$	change in bus voltage angles, in the same order as $\mathbf{\Delta P}$; $(m+n)$ component column vector
$\mathbf{B'}$	matrix containing the negative of susceptance of the $\mathbf{Y}$ matrix; $(m+n)$ x $(m+n)$ matrix
ΔP_{Lij}	changes in losses of the transmission line connecting buses of the ith seller and jth buyer
$\lvert\mathbf{V}_i\rvert$	magnitude of voltage at bus i
δ_i	angle at bus i

$\lvert\mathbf{Y}_{ij}\rvert$	magnitude of the ith row and jth column component of the $\mathbf{Y}$ matrix
γ_{ij}	angle of the ith row and jth column component of the $\mathbf{Y}$ matrix
B_{si}	amount of power submitted by ith seller
B_{bj}	amount of power submitted by jth buyer
lsc	loss coefficient; $(m+n)$ component column vector
$\mathbf{K}$	reduced coefficient from reducing active power flow and active power balance constraints to one constraint, $(m+n)$ component column vector

The spinning and ready reserves are not considered in this thesis. The constraints considered here are active power flow equations, active power balance constraint, and those constraints which specify the bids submitted by GENCOs and DISTCOs. The active power flow equations are used to eliminate the $\boldsymbol{\Delta}\delta_i$ terms in the active power balance constraint and therefore the variables of the formulation are only ΔP_{bj} and ΔP_{si}. The reason that the auction problem is simplified in this manner is that the purpose of this thesis is to show how to implement auctions with various techniques, not to develop the complete formulation for auctions. The general form of the formulation for type 4 auctions is shown as follows:

$$\max_{\Delta P_{bj},\Delta P_{si}} \sum_{j=1}^{n} c_{bj}\Delta P_{bj} - \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.36}$$

subject to active power flow equations (3.37), active power balance constraint (3.39), and bid amount constraints (3.42, 3.43).

All the constraints can be described as below:

active power flow equations

$$\boldsymbol{\Delta}\mathbf{P} - \mathbf{B}'\boldsymbol{\Delta}\delta = \mathbf{0} \tag{3.37}$$

where

$$\boldsymbol{\Delta}\mathbf{P} = [\Delta P_{s1} \; ... \; \Delta P_{si} \; ... \; \Delta P_{sm} \; -\Delta P_{b1} \; ... \; -\Delta P_{bj} \; ... \; -\Delta P_{bn}]^T \tag{3.38}$$

Note that the fast decoupled power flow is used in this thesis so that the $\mathbf{B}'$ is a constant matrix.

active power balance constraint

$$\sum_{i=1}^{m}\Delta P_{si} - \sum_{j=1}^{n}\Delta P_{bj} - \sum_{i=1}^{m}\sum_{j=1}^{n}\Delta P_{Lij} = 0 \tag{3.39}$$

where

$$\Delta P_{Lij} = L_{pij}\Delta\delta_i - L_{pij}\Delta\delta_j \tag{3.40}$$

$$L_{pij} = -|\mathbf{V}_i||\mathbf{V}_j||\mathbf{Y}_{ij}|(sin[-\gamma_{ij} + (\delta_i - \delta_j)] - sin[-\gamma_{ij} - (\delta_i - \delta_j)]) \tag{3.41}$$

In this thesis, the magnitudes and angles of bus voltages in (3.41) are approximated by the original values prior to the auction.

The bid amount constraints combine the bids of sellers or buyers with the non-negativity constraints together, and are shown as follows:

$$0 \leq \Delta P_{si} \leq B_{si}, \qquad i = 1, ..., m \tag{3.42}$$

$$0 \leq \Delta P_{bj} \leq B_{bj}, \qquad j = 1, ..., n \tag{3.43}$$

There are $m+n+1$ constraints for (3.37) and (3.39). These $m+n+1$ constraints can be manipulated to reduce to one constraint as follows:
(3.37) can be rewritten as

$$\mathbf{\Delta\delta} = \mathbf{B'^{-1}\Delta P} \tag{3.44}$$

$\sum_{i=1}^{m}\sum_{j=1}^{n}\Delta P_{Lij}$ is the summation of ΔP_{Lij} of all connections and can be written as

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\Delta P_{Lij} = \mathbf{lsc\Delta\delta} \tag{3.45}$$

where

$$\mathbf{lsc} = [\mathbf{lsc}_1\ \mathbf{lsc}_2\ ...\ \mathbf{lsc}_{m+n}] \tag{3.46}$$

and each component is found by summing the corresponding terms of L_{pij} and $-L_{pij}$ in (3.40). Using (3.45) and (3.44) with some manipulation, the left-hand side of (3.39) in vector form can be rewritten as

$$\mathbf{U\Delta P} = \mathbf{lscB'^{-1}\Delta P} \tag{3.47}$$

where

$$\mathbf{U} = [1\ 1\ ...\ 1] \tag{3.48}$$

To eliminate the negative signs of the last n components of $\mathbf{\Delta P}$, $\mathbf{\Delta P}$ is pre-multiplied by $\mathbf{C}$.

$$\mathbf{\Delta P'} = \mathbf{C\Delta P} \tag{3.49}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{I_m} & 0 \\ 0 & -\mathbf{I_n} \end{bmatrix} \tag{3.50}$$

Using (3.49), (3.47) can be finally written as:

$$\mathbf{K\Delta P'} = \mathbf{0} \tag{3.51}$$

where

$$\mathbf{K} = (\mathbf{U} - \mathbf{lscB'^{-1}})\mathbf{C} \tag{3.52}$$

Thus, the general form of type 4 auction formulation in reduced form can be written as:

$$\max_{\Delta P_{bj}, \Delta P_{si}} \sum_{j=1}^{n} c_{bj}\Delta P_{bj} - \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.53}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} + \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj} = 0$$
$$0 \leq \Delta P_{si} \leq B_{si}, \quad i = 1, ..., m$$
$$0 \leq \Delta P_{bj} \leq B_{bj}, \quad j = 1, ..., n$$

The upper-bounds and lower-bounds of ΔP_{si} and ΔP_{bj} can be shown separately and the formulation for type 4 auctions becomes:

$$\max_{\Delta P_{bj}, \Delta P_{si}} \sum_{j=1}^{n} c_{bj}\Delta P_{bj} - \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.54}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} + \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj} = 0$$
$$\Delta P_{si} \leq B_{si}, \quad i = 1, ..., m$$
$$\Delta P_{bj} \leq B_{bj}, \quad j = 1, ..., n$$
$$\Delta P_{si}, \ \Delta P_{bj}, \geq 0, \ \forall i, j$$

To implement in LR, the maximization problem is changed to minimization problem as follows:

$$\min_{\Delta P_{bj},\Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} - \sum_{j=1}^{n} c_{bj}\Delta P_{bj} \tag{3.55}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} + \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj} = 0 \tag{3.56}$$

$$\Delta P_{si} \leq B_{si}, \quad i = 1, ..., m \tag{3.57}$$

$$\Delta P_{bj} \leq B_{bj}, \quad j = 1, ..., n \tag{3.58}$$

$$\Delta P_{si}, \geq 0, \ \forall i, \tag{3.59}$$

$$\Delta P_{bj}, \geq 0, \ \forall j, \tag{3.60}$$

3.1.3.2 Formulation of LR-based type 4 auctions

By relaxing the coupling constraint, (3.56), the LaGrange function, L, is shown in (3.61) with LaGrange multiplier, λ:

$$L = \sum_{i=1}^{m} c_{si}\Delta P_{si} - \sum_{j=1}^{n} c_{bj}\Delta P_{bj} + \lambda\left(-\sum_{i=1}^{m} \mathbf{K}_i\Delta P_{si} - \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj}\right) \tag{3.61}$$

The local constraints, (3.57) to (3.60), can also be relaxed by adding them with additional LaGrange multipliers, μ_{si}, ν_{si}, μ_{bj}, and ν_{bj} and the LaGrange function becomes:

$$\begin{aligned} L \ = \ & \sum_{i=1}^{m} c_{si}\Delta P_{si} - \sum_{j=1}^{n} c_{bj}\Delta P_{bj} + \lambda\left(-\sum_{i=1}^{m} \mathbf{K}_i\Delta P_{si} - \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj}\right) \\ & + \sum_{i=1}^{m} \mu_{si}(\Delta P_{si} - B_{si}) + \sum_{j=1}^{n} \mu_{bj}(\Delta P_{bj} - B_{bj}) \\ & - \sum_{i=1}^{m} \nu_{si}\Delta P_{si} - \sum_{j=1}^{n} \nu_{bj}\Delta P_{bj} \end{aligned} \tag{3.62}$$

The derivative of L with respect to ΔP_{si} and ΔP_{bj} can be derived and is shown in (3.63), and (3.64).

$$\frac{\partial L}{\partial \Delta P_{si}} = c_{si} - \mathbf{K}_i\lambda + \mu_{si} - \nu_{si} \tag{3.63}$$

$$\frac{\partial L}{\partial \Delta P_{bj}} = -c_{bj} - \mathbf{K}_{m+j}\lambda + \mu_{bj} - \nu_{bj} \tag{3.64}$$

At optimality, $\frac{\partial L}{\partial \Delta P_{si}}$ and $\frac{\partial L}{\partial \Delta P_{bj}}$ are equal to zero. Thus, (3.63) becomes (3.65) and (3.64) becomes (3.66).

$$\lambda = \frac{c_{si}}{\mathbf{K}_i} + \frac{\mu_{si}}{\mathbf{K}_i} - \frac{\nu_{si}}{\mathbf{K}_i} \tag{3.65}$$

$$\lambda = \frac{c_{bj}}{-\mathbf{K}_{m+j}} - \frac{\mu_{bj}}{-\mathbf{K}_{m+j}} + \frac{\nu_{bj}}{-\mathbf{K}_{m+j}} \tag{3.66}$$

(3.65) and (3.66) will be used for step two of the algorithm described in subsection 3.1.3.3. Note that there are many LaGrange multipliers in (3.62), but only the coupling constraint LaGrange multiplier, λ, will be used in the iterations of the algorithm described in subsection 3.1.3.3. Note that $\mathbf{K}_{m+j}$ is negative and therefore $-\mathbf{K}_{m+j}$ is positive.

3.1.3.3 Algorithm

Because the formulation for type 4 auctions is not very complex, the procedure of using LR to implement the auctions is reduced to a simple algorithm as shown in Fig. 3.5, rather than continuously switching between solving the primal and dual problems. Steps one and eight of the algorithm will be explained in subsection 3.1.3.4. Note that the maximum number of iterations, $iter^{max}$, is set to 20. In step two of the algorithm, ΔP_{si} and ΔP_{bj} are determined by considering the curves in Fig. 3.6 and Fig. 3.7. Based on (3.65) and (3.66), the curves of λ versus ΔP_{si} and ΔP_{bj} can be plotted in Fig. 3.6 and Fig. 3.7. Fig. 3.6 illustrates the value of $\frac{\mu_{si}}{\mathbf{K}_i}$ when λ is equal to λ_1 in which ν_{si} is equal to zero. Fig. 3.7 illustrates the value of $\frac{\mu_{bj}}{-\mathbf{K}_{m+j}}$ when λ is equal to λ_2 in which ν_{bj} is equal to zero.

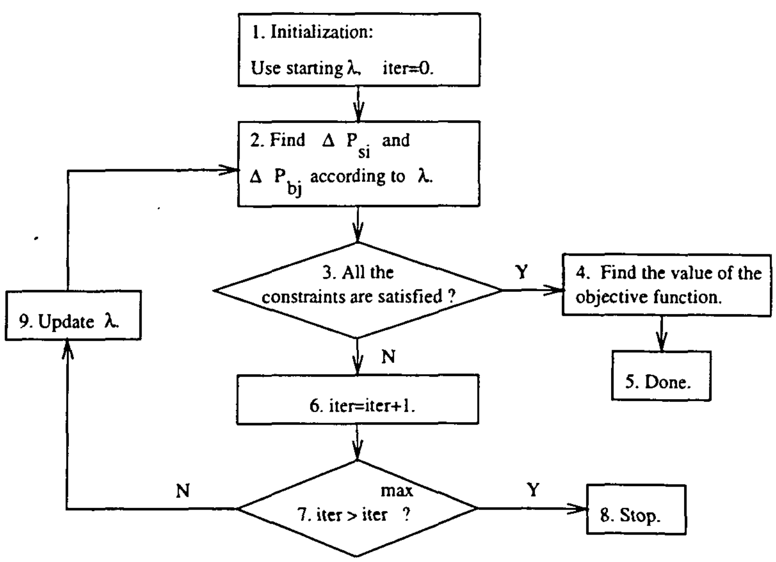

Figure 3.5 Algorithm for LR-based type 4 auctions

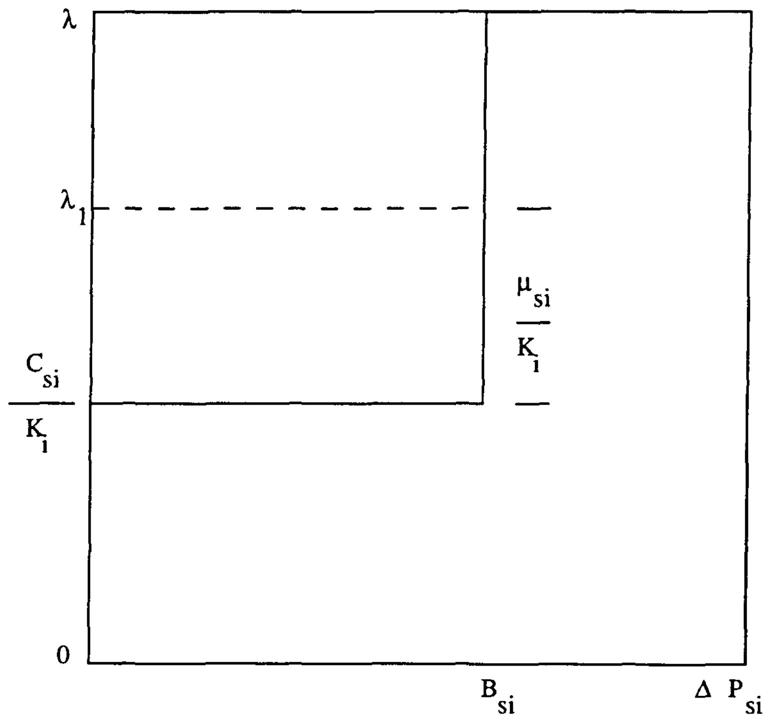

Figure 3.6 Curve of λ versus ΔP_{si}

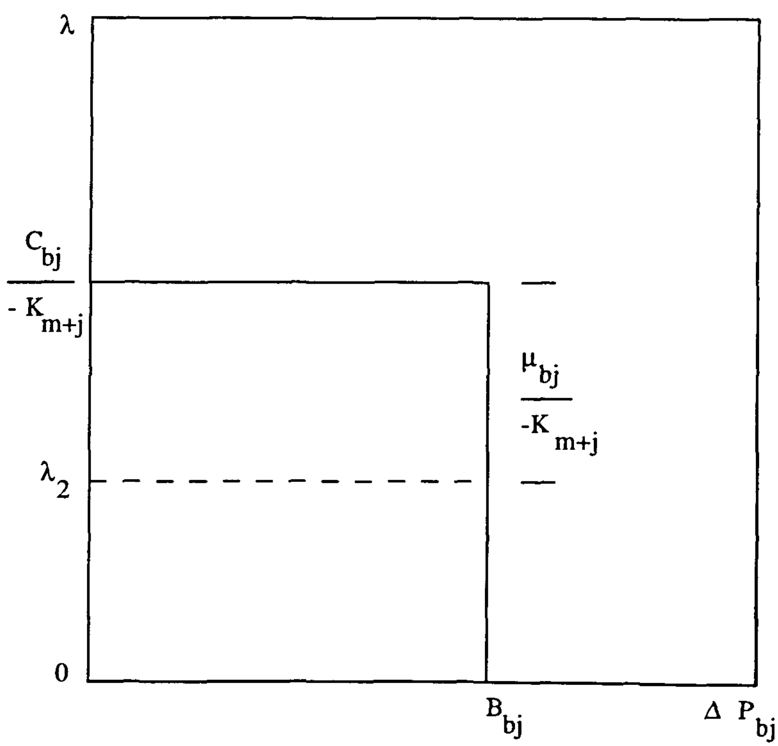

Figure 3.7 Curve of λ versus ΔP_{bj}

From Fig. 3.6, the value of ΔP_{si} can be determined to be as follows:

$$\Delta P_{si} = B_{si} \qquad if\ \lambda > \frac{c_{si}}{\mathbf{K}_i} \tag{3.67}$$

$$\Delta P_{si} = 0 \qquad if\ \lambda < \frac{c_{si}}{\mathbf{K}_i} \tag{3.68}$$

Similarly, from Fig. 3.7, the value of ΔP_{bj} can be determined to be as follows:

$$\Delta P_{bj} = B_{bj} \qquad if\ \lambda < \frac{c_{bj}}{-\mathbf{K}_{m+j}} \tag{3.69}$$

$$\Delta P_{bj} = 0 \qquad if\ \lambda > \frac{c_{bj}}{-\mathbf{K}_{m+j}} \tag{3.70}$$

Due to the configuration of this problem, (3.55) to (3.60), there will be $m+n-1$ variables binding, i.e. they are equal to 0, B_{si}, or B_{bj}, and therefore the value of the other variable can be determined from (3.56). This is true except only at the worse case, for some values of λ, when all $m+n$ variables are binding and (3.56) cannot be satisfied. The optimal λ will be equal to $\frac{c_{si}}{\mathbf{K}_i}$ if the non-binding variable belongs to the sellers or $\frac{c_{bj}}{-\mathbf{K}_{m+j}}$ if the non-binding variable belongs to the buyers.

Similar to subsection 3.1.2.1, the optimal λ can be found from the intersection of the aggregate λ curve of sellers and the aggregate λ curve of buyers. However, for this problem, the aggregate λ curve of sellers is not plotted versus $\sum_{i=1}^{m} \Delta P_{si}$ but is plotted versus $\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si}$. The aggregate λ curve of buyers is also not plotted versus $\sum_{i=1}^{n} \Delta P_{bj}$ but is plotted versus $\sum_{j=1}^{n} \mathbf{K}_{m+j} \Delta P_{bj}$. The two curves are plotted on the common horizontal axis. This concept will be illustrated along with the result in subsection 4.1.4.

3.1.3.4 Updating procedure

As stated in subsection 3.1.3.3, the optimal λ will be equal to $\frac{c_{si}}{\mathbf{K}_i}$ if the non-binding variable belongs to the sellers or $\frac{c_{bj}}{-\mathbf{K}_{m+j}}$ if the non-binding variable belongs to the buyers. Thus, all the $\frac{c_{si}}{\mathbf{K}_i}$ of sellers and all the $\frac{c_{bj}}{-\mathbf{K}_{m+j}}$ of buyers are enumerated and sorted from the lowest value to the highest value. The algorithm starts by taking the lowest value as the starting λ and uses the next greater value to be the new λ in each iteration.

3.1.4 Applying LR to type 3 auctions

In type 3 auctions, all GENCOs and IPPs submit their bids to the ICA, and DISTCOs submit their hourly loads to the ICA. Then the ICA implements the auction so that the total revenue of GENCOs and IPPs is minimized.

3.1.4.1 General type 3 auction formulation

The general formulation for type 3 auctions can be modified from that of type 4 auctions, (3.55) to (3.60), and is shown as follows:

$$\min_{\Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.71}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} = F \tag{3.72}$$

$$\Delta P_{si} \leq B_{si}, \quad i = 1, ..., m \tag{3.73}$$

$$\Delta P_{si} \geq 0, \ \forall i \tag{3.74}$$

where

$$F = -\sum_{j=1}^{n} \mathbf{K}_{m+j} \Delta P_{bj} \tag{3.75}$$

which is a constant because in type 3 auctions which are single-sided auctions, ΔP_{bj} is the change in load at each load bus.

3.1.4.2 Formulation of LR-based type 3 auctions

The coupling constraint, (3.72) and the local constraints, (3.73) and (3.74) can be relaxed and the LaGrange function can be written as:

$$\begin{aligned} L \ = \ & \sum_{i=1}^{m} c_{si}\Delta P_{si} + \lambda(F - \sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si}) \\ & + \sum_{i=1}^{m} \mu_{si}(\Delta P_{si} - B_{si}) - \sum_{i=1}^{m} \nu_{si}\Delta P_{si} \end{aligned} \tag{3.76}$$

The derivative of L with respect to ΔP_{si} can be derived and is shown in (3.77).

$$\frac{\partial L}{\partial \Delta P_{si}} = c_{si} - \mathbf{K}_i \lambda + \mu_{si} - \nu_{si} \tag{3.77}$$

At optimality, $\frac{\partial L}{\partial \Delta P_{si}}$ is equal to zero. Thus, (3.77) becomes (3.78).

$$\lambda = \frac{c_{si}}{\mathbf{K}_i} + \frac{\mu_{si}}{\mathbf{K}_i} - \frac{\nu_{si}}{\mathbf{K}_i} \tag{3.78}$$

Note that (3.77) and (3.78) are the same as (3.63) and (3.65) of LR-based type 4 auctions.

3.1.4.3 Algorithm

The algorithm for LR-based type 3 auctions is the same as that for LR-based type 4 auctions, as such, Fig. 3.6 applies to type 3 auctions; ΔP_{si} can be determined from (3.67) and (3.68). Similar to type 4 auctions, there will be $m-1$ binding variables, i.e. equal to 0 or B_{si}, and so the value of the other variable can be determined from (3.72).

The optimal λ can be found from the value of λ of the aggregate λ curve of sellers at the value F of the horizontal axis. The horizontal axis of the aggregate λ curve of sellers is $\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si}$. This concept will be illustrated along with the result in subsection 4.1.3.

3.1.4.4 Updating procedure

The concept and procedure are the same as those of LR-based type 4 auctions except that $\frac{c_{bj}}{-\mathbf{K}_{m+j}}$ of buyers does not exist, so $\frac{c_{bj}}{-\mathbf{K}_{m+j}}$ is not considered in the updating procedure of LR-based type 3 auctions.

3.1.5 Implementation problems in applying LR to type 1 and 2 auctions

Some utilities have already adopted the LR-based auctions, which are equivalent to type 1 auctions in this thesis for trading power. There will be many independent power producers (IPP) in the new competitive market and therefore identical or similar generating units will be prevalent in the market. This prevents us from handling the identical units as they were handled in Virmani et al. [13] and Slawaji et al. [18]. Adjusting the heat rates may not be done due to the fairness issue and the solution found by committing units as a group may not be the optimal solution for the system.

The problems studied in this thesis are divided into two categories, problems with identical units and problems with similar units. For identical units, LR will always select or deny all the identical units simultaneously no matter what the optimal solution is. This means that LR will probably be unable to find the optimal solution and sometimes not be able to even find a feasible solution. For similar units, sometimes the optimal solution requires selection of only some of these units. Any subsets of similar units can be selected for the optimal solution. However, not all units may be selected as this would cause overgeneration. This is inequitable to the unchosen units which actually could provide an alternative optimal solution. The case-studies showing these problems will be described in subsection 4.1.5.

Although the problems stated above also occur in UC, they are more intense when LR is used for implementing an auction due to two main reasons. First, auction has been proposed for use in the deregulated environment which will have many identical or similar units and these units are the cause of the problems discussed here. Second, auctions are very dynamic and they change every period, so the proper algorithm and the LaGrange multiplier updating procedure that can be used for one period may be inappropriate for use in other periods.

3.2 Interior-Point Linear Programming (IPLP)

3.2.1 Algorithm of IPLP

Various algorithms have been developed for IPLP. In this thesis the IPLP method used to implement auctions is the affine-scaling primal algorithm. Also, explanation of IPLP is coupled with the affine-scaling primal algorithm. This algorithm is chosen because it is simple and also efficient. The explanation of IPLP's algorithm is divided into two parts. The first part gives the basic concept of the affine-scaling primal algorithm. The explanation of the basic formulation and the algorithm is based on Arbel [35]. The basic formulation is explained in the first part and the basic algorithm is described along with the developed algorithm in the second part to be the complete algorithm of IPLP used in this thesis. The developed algorithm in the second part comes from the addition of a section to the basic algorithm so that IPLP can find an exact solution.

3.2.1.1 Basic concept

Unlike the simplex method, IPLP reaches a solution by moving through the interior of feasible region. Two major components of the affine-scaling primal algorithm are centering and projective gradient direction. Movement is made through projective gradient direction for maximizing the objective function or opposite to projective gradient direction for minimizing objective function. The projective gradient direction is used instead of gradient direction for the purpose of maintaining feasibility. Centering is performed to achieve the potential to improve objective function in each iteration. Centering is made through the following procedure. In each iteration, the linear program and the current solution vector are scaled so that the components of the scaled solution vector have equal distances from all the edges of the scaled feasible region. Then the current scaled solution vector is updated to the new solution vector and the new solution vector is rescaled back to the original space. The operation used to perform rescaling is affine transformation. This is why this class of IPLP algorithm is called affine-scaling algorithm. In addition, the scaling and rescaling processes are built in the algorithm so that all of the steps in the algorithm are performed in the original space.

3.2.1.1.1 Starting solution The starting solution must lie within the feasible region. It is not always be easy to find such a starting solution. Thus, the original problem is augmented to become a problem in which any $\mathbf{P_0}$, having all components greater than zero can be part of $\mathbf{P'}$ to be the starting solution of the augmented problem. This thesis uses vector one, [1 1 ... 1], as the starting solution vector, $\mathbf{P'}$. The big M method is used to construct the augmented problem. The augmented problem has one additional variable. If this additional variable is driven to zero at the end of the algorithm, the primal problem is feasible. Otherwise, the primal problem is infeasible.

The original and augmented problems are shown at (3.79) and (3.80) respectively. Note that M is a big positive number and β is a m-component vector.

Original linear program

$$\min_{\mathbf{P}} \mathbf{c}^T \mathbf{P} \tag{3.79}$$

subject to

$$\mathbf{AP} = \mathbf{b}$$

$$\mathbf{P} \geq \mathbf{0}$$

where

$\mathbf{P}$: variables; n-component column vector

$\mathbf{c}^T$: cost coefficients; n-component row vector

$\mathbf{A}$: technological coefficients; m x n matrix

$\mathbf{b}$: right hand side; m-component column vector

Augmented linear program

$$\min_{\mathbf{P'}} \mathbf{c'}^T \mathbf{P'} \tag{3.80}$$

subject to

$$\mathbf{A'P'} = \mathbf{b}$$

$$\mathbf{P'} \geq \mathbf{0}$$

where

$$\mathbf{c'} = [\mathbf{c}^T \ M]^T \tag{3.81}$$

$$\mathbf{P}' = [\mathbf{P}_0^T \ 1]^T \tag{3.82}$$

$$\mathbf{A}' = [\mathbf{A} \ \beta] \tag{3.83}$$

$$\beta = \mathbf{b} - \mathbf{A}\mathbf{P}_0 \tag{3.84}$$

3.2.1.1.2 Stopping criteria Three quantities, duality gap, primal feasibility, and dual feasibility, are used as the stopping criteria for terminating the algorithm. These quantities are defined in (3.85), (3.86), and (3.87); $\mathbf{y}$ and $\mathbf{z}$ are defined in (3.89) and (3.90) respectively. If all three criteria are met simultaneously, the algorithm is terminated. In other words, the solution found is very close to the optimal solution. This type of solution is called ϵ-optimal solution. Although dual feasibility is desired in terminating the algorithm, sometimes it cannot be achieved. In the used algorithm IPLP in this thesis, the dual feasibility stopping criterion is neglected and the algorithm still works quite well with many test problems.

$$duality\ gap = \frac{norm(\mathbf{c}'^T\mathbf{P}' - \mathbf{b}^T\mathbf{y})}{1 + norm(\mathbf{c}'^T\mathbf{P}')} \tag{3.85}$$

$$primal\ feasibility = \frac{norm(\mathbf{b} - \mathbf{A}'\mathbf{P}')}{1 + norm(\mathbf{b})} \tag{3.86}$$

$$dual\ feasibility = \frac{\mathbf{c}' - \mathbf{A}'^T\mathbf{y} - \mathbf{z}}{1 + norm(\mathbf{c}')} \tag{3.87}$$

3.2.1.2 Developed algorithm

In the basic algorithm, IPLP can find only the ϵ-optimal solution, but not the exact optimal solution, because the solution found by IPLP is still inside the feasible region. The closer to the exact solution, the smaller the values of the stopping criteria are required, which might cause numerical instability.

In this thesis, the IPLP is developed to be able to find the exact optimal solution. In other words, the IPLP can reach the optimal vertex (extreme point). The main concept can be explained as follows: a quantity, $\mathbf{z}$ (defined at (3.90)), is calculated at every iteration and $\mathbf{z}$ is the estimate of the reduced cost coefficient vector. When the current solution is very close to the optimal vertex, the components of $\mathbf{z}$ which belong to the basic variables of the optimal vertex are very close to zero. Using this concept, the algorithm can check to see if the duality gap and primal feasibility are satisfied and ensure that the number of components of $\mathbf{z}$ which are very close to zero is equal to the number of constraints. The estimated optimal basic variables are the variables having satisfied values of $\mathbf{z}$. These estimated optimal basic variables can be verified with the Karush-Kuhn-Tucker (KKT) conditions for optimality.

The KKT conditions can be seen in Bazaraa et al. [45], page 221-227. If the KKT conditions are satisfied, the estimated optimal basic variables are correct and the optimal solution can be calculated.

Apart from the fact that this algorithm can find the exact solution, the great benefit of the algorithm developed in this thesis is that sensitivity analysis can be performed after the optimal solution is found.

The developed algorithm for IPLP, affine-scaling primal algorithm, in this thesis is described as follows:

Step 0:

Initialize iteration counter, $iter = 0$.

Initialize ϵ_1 and ϵ_2 as 1e-4 and 1e-6 respectively.

Initialize the starting solution vector, $\mathbf{P}'(iter)$=[1 1 ... 1]^T.

Step 1:

Increment the iteration counter, $iter = iter + 1$.

Define the scaling matrix $\mathbf{D}(iter)$ by

$$\mathbf{D}(iter) = diag(\ [\mathbf{P'}_1(iter)\ \mathbf{P'}_2(iter)\ \ldots\ \mathbf{P'}_{n+1}(iter)]\ ), \tag{3.88}$$

where $diag(\mathbf{P}')$ means diagonal matrix of vector $\mathbf{P}'$ and $\mathbf{P'}_i(iter)$ is the ith component of the current $\mathbf{P}'(iter)$.

Step 2:

Calculate the dual estimate, $\mathbf{y}(iter)$, where $\mathbf{y}(iter)$ is a m-component column vector, by solving

$$[\mathbf{A}'\mathbf{D}^2(iter)\mathbf{A}'^T][\mathbf{y}(iter)] = [\mathbf{A}'\mathbf{D}^2(iter)\mathbf{c}']. \tag{3.89}$$

Step 3:

Find the estimate of the reduced cost vector, $\mathbf{z}(iter)$ and then use it to find the primal step direction vector, $\mathbf{dP}'(iter)$, where $\mathbf{z}(iter)$ and $\mathbf{dP}'(iter)$ are $n + 1$ component column vectors, by

$$\mathbf{z}(iter) = \mathbf{c}' - \mathbf{A}'^T\mathbf{y}(iter), \tag{3.90}$$

$$\mathbf{dP}'(iter) = -\mathbf{D}^2(iter)\mathbf{z}(iter). \tag{3.91}$$

Step 4:

Update the solution vector by

$$\mathbf{P}'(iter + 1) = \mathbf{P}'(iter) + \rho\alpha\mathbf{dP}'(iter), \tag{3.92}$$

$$\alpha = min\{\frac{-\mathbf{P'}_i(iter)}{\mathbf{dP'}_i(iter)} : \forall \mathbf{dP'}_i(iter) < 0,\ 1 \leq i \leq n+1\}. \tag{3.93}$$

α is the maximum allowable step size which maintains feasibility and changing by step size α will make at least one variable hit the boundary of the feasible region. Thus, a factor ρ is used to make the new solution remain inside the feasible region. The algorithm uses ρ as 0.95 for the results presented in this thesis.

Step 5:

Test with two criteria.

First criterion: duality gap and primal feasibility are less than ϵ_1.

Second criterion: the number of components of $\mathbf{z}$ which are less than ϵ_2 is equal to the number of constraints.

If both criteria are satisfied, go to step 6. Otherwise, go to step 1.

Step 6:

The expected optimal basic variables are the variables having $\mathbf{z}$ less than ϵ_2. Test with the KKT conditions. If the KKT conditions are satisfied, the optimal basic variables have been found, go to step 7. If not, go to step 5 and reduce ϵ_1 and ϵ_2 to be one-tenth of the value previously used in step 5.

Step 7:

Find the optimal solution from

$$\mathbf{P'_B} = \mathbf{B}^{-1}\mathbf{b} \tag{3.94}$$

where $\mathbf{P'_B}$ contains values of basic variables. Values of non-basic variables are zero. The value of objective function can be calculated from substituting the values of all the decision variables into the objective function.

Step 0 to step 4 of the algorithm is based on Arbel [35], and step 5 to step 7 of the algorithm is developed by the author. The developed algorithm also can check for infeasibility, unboundedness, and degeneracy of primal problem. The result is infeasible if the selected variables at step 5 contain the artificial variable added for the augmented problem. The result is unboundedness if for any iteration, all of the components of $\mathbf{dP'}(iter)$ found at step 3 are all greater than or equal to zero. The result is degenerate if the reduced cost coefficient of any of the non-basic variables while testing with the KKT conditions at step 6 is calculated to be zero.

At step 6, ϵ_1 and ϵ_2 are repeatedly reduced to be one-tenth if the KKT conditions are still not satisfied. Actually this situation is unlikely because the original values of ϵ_1 and ϵ_2 (1e-4 and 1e-6 respectively) are set reasonably. These values are tested with many examples and the test shows that these values can be used as criteria for terminating the algorithm's interior point part (step 0 to step 4) to find the exact solution at step 7.

3.2.2 Applying IPLP to type 3 auctions

The IPLP technique is used to implement type 3 and 4 auctions in this thesis. The linear program for IPLP-based type 3 auctions is the same as the general form of formulation of LR-based type 3 auctions, (3.71) to (3.75). The upper-bounds of ΔP_{si}, B_{si} are implemented as normal constraints and the lower-bounds are implemented as non-negativity constraints. The formulation can be reshown as follows:

$$\min_{\Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.95}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} = F$$
$$\Delta P_{si} \leq B_{si}, \quad i = 1, \ldots, m$$
$$\Delta P_{si} \geq 0, \ \forall i$$

where

$$F = -\sum_{j=1}^{n} \mathbf{K}_{m+j} \Delta P_{bj} \tag{3.96}$$

F is a constant because ΔP_{bj} is change in load at each load bus.

3.2.3 Applying IPLP to type 4 auctions

The linear program for IPLP-based type 4 auctions is the same as the general form of formulation of LR-based type 4 auctions, (3.55) to (3.60). To implement IPLP, the upper-bounds of ΔP_{si}, B_{si} and the upper-bounds of ΔP_{bj}, B_{bj} are implemented as normal constraints and the lower-bounds are implemented as non-negativity constraints. The linear program for type 4 auctions becomes:

$$\min_{\Delta P_{bj}, \Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} - \sum_{j=1}^{n} c_{bj}\Delta P_{bj} \tag{3.97}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} + \sum_{j=1}^{n} \mathbf{K}_{m+j} \Delta P_{bj} = 0$$

$$\Delta P_{si} \leq B_{si}, \quad i = 1, ..., m$$

$$\Delta P_{bj} \leq B_{bj}, \quad j = 1, ..., n$$

$$\Delta P_{si}, \geq 0, \ \forall i,$$

$$\Delta P_{bj}, \geq 0, \ \forall j,$$

3.3 Upper-Bound Linear Programming (UBLP)

This algorithm is useful for the linear program having variables with upper-bounds ($0 \leq \mathbf{P}_i \leq \mathbf{P}_i^{max}$). In the type 3 and 4 auction problems , the variables have the upper-bounds in this form. This is why UBLP is used to implement type 3 and 4 auctions in this thesis. The UBLP algorithm has few differences from the simplex algorithm. The major difference in concept is that instead of implementing the upper-bounds of variables as the normal constraints of the standard form, the upper-bounds of variables are treated as the same type of the non-negativity constraints. In this way the dimension of the constraints are greatly reduced which helps to reduce the computing and storage requirements. To show how the dimension of the constraints are reduced, the following linear programs, (3.98) and (3.99) are shown.

The linear program with variables having upper-bounds in the standard form is shown in (3.98). The UBLP uses (3.98) in implementation in which the dimension of the constraints is the dimension of matrix A, assumed as m x n. If problem (3.98) is implemented using the simplex method, (3.98) is modified to the standard form problem, (3.99). Dimension of the constraints of problem (3.99) is $(m + n)$ x $2n$, which is much greater than the dimension of constraints of (3.98), m x n.

$$\min_{\mathbf{P}} \mathbf{c}^T \mathbf{P} \tag{3.98}$$

subject to

$$\mathbf{AP} = \mathbf{b}$$

$$0 \leq \mathbf{P}_i \leq \mathbf{P}_i^{max}$$

$$\min_{\mathbf{P}} \mathbf{c}^T \mathbf{P} \tag{3.99}$$

subject to

$$\mathbf{AP} = \mathbf{b}$$

$$\mathbf{P} + \mathbf{s} = \mathbf{P}^{\mathbf{max}}$$

$$\mathbf{P} \geq \mathbf{0} \qquad \mathbf{s} \geq \mathbf{0}$$

where

$\mathbf{P}$: variables; n-component column vector

$\mathbf{c}^T$: cost coefficients; n-component row vector

$\mathbf{A}$: technological coefficients; (m x n) matrix

$\mathbf{b}$: right hand side; m-component column vector

$\mathbf{s}$: slack variables; m-component column vector

The algorithm used to build the computer program for UBLP to implement type 3 and 4 auctions is based on Luenberger [47]. The algorithm is very similar to that of the simplex method except that all of the non-basic variables of the simplex method are zero (lower bound of UBLP problem), but the non-basic variables of UBLP can be at the lower bounds, 0, or at the upper bound, $\mathbf{P}_i^{max}$. If the non-basic variable is at the lower bound, it is $\mathbf{P}_i^-$ in the tableau, and $\mathbf{P}_i^- = \mathbf{P}_i$. If the non-basic variable is at the upper bound, it is $\mathbf{P}_i^+$ in the tableau, and $\mathbf{P}_i^+ = \mathbf{P}_i^{max} - \mathbf{P}_i$. As the iterations progress, the non-basic variable is changed back and forth from $\mathbf{P}_i^-$ to $\mathbf{P}_i^+$. In the program, there is a variable, $\mathbf{bnd}_i$, to indicate that the non-basic variable is at the lower or upper bound. If the non-basic variable is at the lower bound, $\mathbf{bnd}_i = 0$. If the non-basic variable is at the upper bound, $\mathbf{bnd}_i = 1$. The algorithm is summarized as follows:

Step 1:

Find a starting basic feasible solution.

Step 2:

Find the non-basic variable which has the most positive reduced-cost coefficient. (Assume variable $\mathbf{P}_k$ is selected.) If all the reduced-cost coefficients of non-basic variables are less than or equal to zero, the current solution is optimal solution. Stop.

Step 3:

Calculate three numbers, R_1 from (3.100) or (3.101), R_2 from (3.102), and R_3 from (3.103).

$$R_1 = \mathbf{P}_k^{max} \quad if\ \mathbf{bnd}_k = 0 \tag{3.100}$$

$$R_1 = 0 \quad if\ \mathbf{bnd}_k = 1 \tag{3.101}$$

$$R_2 = \min \frac{\mathbf{b}_i^{iter}}{\mathbf{y}_{ik}}, \quad \forall \mathbf{y}_{ik} > 0 \tag{3.102}$$

$$R_3 = \min \frac{\mathbf{b}_i^{iter} - \mathbf{P}_k^{max}}{\mathbf{y}_{ik}}, \quad \forall \mathbf{y}_{ik} < 0 \tag{3.103}$$

where

$\mathbf{y}_{ik}$: current technological coefficient at ith row and kth column

$\mathbf{b}^{iter}$: current right hand side

Step 4:

Choose the minimum number among R_1, R_2, and R_3 and then update the tableau as either of the following three cases:

R_1 is chosen: The variable $\mathbf{P}_k$ is changed to its opposite bound.

Subtract $\mathbf{P}_k^{max}$ times column k from right hand side, $\mathbf{b}^{iter}$.

Multiply column k (including its reduced cost coefficient) by -1 and reverse $\mathbf{bnd}_k$.

The basis does not change. This case does not require pivot.

R_2 is chosen: The basic variable of the pivoting row returns to its old bound.

Pivot in the same manner as the simplex method.

R_3 is chosen: The basic variable of the pivoting row is changed to the opposite of its old bound.

Subtract $\mathbf{P}_k^{max}$ from $\mathbf{b}_k^{iter}$.

Reverse the sign of $\mathbf{y}_{rk}$ and reverse $\mathbf{bnd}_k$. r is the pivot row.

Go to step 2.

Note that in the computer program, only the technological coefficients and reduced-cost coefficients of non-basic variables are stored in each iteration during the procedure. This helps reduce storage requirements while running the program.

3.3.1 Applying UBLP to type 3 auctions

The linear program is the same as (3.95) of subsection 3.2.2 in the IPLP section except that instead of implementing the upper-bounds of the bids as the normal constraints, they are implemented in the same manner as the non-negativity constraints. The linear program can be shown below:

$$\min_{\Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} \tag{3.104}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} = F$$
$$0 \le \Delta P_{si} \le B_{si}, \qquad i = 1, ..., m$$

where

$$F = -\sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj} \tag{3.105}$$

F is a constant because ΔP_{bj} is change in load at each load bus.

3.3.2 Applying UBLP to type 4 auctions

Instead of implementing the upper-bounds of the bids as the normal constraints as (3.97) of subsection 3.2.3 in the IPLP section, they are implemented as the same manner as the non-negativity constraints. The linear program is reshown below:

$$\min_{\Delta P_{bj}, \Delta P_{si}} \sum_{i=1}^{m} c_{si}\Delta P_{si} - \sum_{j=1}^{n} c_{bj}\Delta P_{bj} \tag{3.106}$$

subject to

$$\sum_{i=1}^{m} \mathbf{K}_i \Delta P_{si} + \sum_{j=1}^{n} \mathbf{K}_{m+j}\Delta P_{bj} = 0$$
$$0 \le \Delta P_{si} \le B_{si}, \qquad i = 1, ..., m$$
$$0 \le \Delta P_{bj} \le B_{bj}, \qquad j = 1, ..., n$$

4 RESULTS AND DISCUSSION

This chapter presents the results of the illustrative auction examples implemented with LR, IPLP, and UBLP. For type 1 and 2 auctions, the network constraints are not considered. For type 3 and 4 auctions, a six-bus system is used to demonstrate the implementation of network constraints. The system data and a figure of the six-bus system are contained in the Appendix.

4.1 LaGrangian Relaxation (LR)

For ease of notation, starting λ, which is composed of four elements, (from the first to fourth stages), will be symbolized as shown in Table 4.1. These starting λ will be used in subsections 4.1.1, 4.1.2, and 4.1.5.

Table 4.1 Reference notation for λ

Notation	λ
λ_a	[12.5 12.5 12.5 12.5]
λ_b	[6 6 6 6]
λ_c	[7.7 9.8 16.3 14.2]
λ_d	[9 9 9 9]
λ_e	[6 6 12.5 12.5]
λ_f	[6 6 12.5 6]
λ_g	[6 12.5 12.5 6]

For subsections 4.1.1, 4.1.2, and 4.1.5, the auctions in the illustrative examples are performed in only four stages, one hour per stage; thus, the minimum up and down time constraints are neglected. Note that the data of GENCOs used in this section (4.1) and the load data used in subsections 4.1.1, 4.1.2, 4.1.5, and 4.1.6 are based on the data of a UC example in Wood et al. [34].

4.1.1 Applying LR to type 1 auctions

There are three GENCOs in this example, each with one generating unit. To relate this example to the example in subsection 4.1.5, the start-up costs are neglected. The GENCOs' data is shown in

Table 4.2 GENCO data for subsection 4.1.1

GENCO i	a_{ig}	b_{ig}	c_{ig}	P_{ig}^{min}	P_{ig}^{max}
GENCO 1	0.0020	10	500	100	600
GENCO 2	0.0025	8	300	100	400
GENCO 3	0.0050	6	100	50	200

Table 4.3 Load data for subsection 4.1.1

Stage	1	2	3	4
Load	170	520	1100	330

Table 4.4 Optimal solution for subsection 4.1.1

Stage(t)	GENCO 1	GENCO 2	GENCO 3
1	0	0	170
2	0	320	200
3	500	400	200
4	0	130	200

Table 4.5 Optimal GENCO costs for all stages for subsection 4.1.1

GENCO i	GENCO 1	GENCO 2	GENCO 3
Cost	6000.00	8398.25	5764.50

Table 4.2 and the load data for the four stages is shown in Table 4.3. The optimal solution of the auction is shown in Table 4.4. The optimal production cost of each GENCO for all stages is shown in Table 4.5. The total optimal production cost is \$20162.75.

The solution indicates that GENCO 3 is selected first. GENCOs 1 and 2 are selected only when GENCO 3 cannot supply the load and GENCO 1 is selected when GENCO 2 cannot supply the load. This is true because GENCO 3 has the least expensive generation and GENCO 1 has the most expensive generation.

4.1.2 Applying LR to type 2 auctions

There are three GENCOs and three DISTCOs in this example. The three GENCOs are the same as the three GENCOs in subsection 4.1.1. The data of the three DISTCOs changes based upon the stages and is shown in Table 4.6, 4.7, 4.8, and 4.9. The start-up costs are again neglected as in subsection 4.1.1.

The optimal solution is shown in Table 4.10. The optimal total cost for each GENCO and the

Table 4.6 DISTCO data at t=1 for subsection 4.1.2

DISTCO i	a_{id}	b_{id}	c_{id}	L_i^{min}	L_i^{max}
DISTCO 1	-0.0025	8.55	350	50	200
DISTCO 2	-0.0040	6.50	50	50	150
DISTCO 3	-0.0055	6.50	50	50	150

Table 4.7 DISTCO data at t=2 for subsection 4.1.2

DISTCO i	a_{id}	b_{id}	c_{id}	L_i^{min}	L_i^{max}
DISTCO 1	-0.0015	12.05	450	50	350
DISTCO 2	-0.0035	10.79	300	50	200
DISTCO 3	-0.0050	7.00	50	50	150

Table 4.8 DISTCO data at t=3 for subsection 4.1.2

DISTCO i	a_{id}	b_{id}	c_{id}	L_i^{min}	L_i^{max}
DISTCO 1	-0.0010	14.90	500	200	600
DISTCO 2	-0.0020	14.00	300	150	350
DISTCO 3	-0.0025	12.75	100	50	250

Table 4.9 DISTCO data at t=4 for subsection 4.1.2

DISTCO i	a_{id}	b_{id}	c_{id}	L_i^{min}	L_i^{max}
DISTCO 1	-0.0020	10.20	400	50	250
DISTCO 2	-0.0030	9.13	250	50	150
DISTCO 3	-0.0050	8.00	50	50	150

optimal total revenue for each DISTCO for all stages are shown in Table 4.11. The difference between the optimal total revenue of DISTCOs and the optimal total production cost of GENCOs is \$8042.9. The optimal price from EDC for each stage is shown in Table 4.12. From the optimal solution, it can be seen that the total system loads that can be supplied in stages 1, 2, 3, and 4 are 170, 520, 1100, and 200 respectively. If any DISTCOs cannot buy enough power to supply their expected loads, they will modify their revenue curve functions prior to submitting them to the ICA next time. Similarly, if any GENCOs cannot sell the power they expected, they will also modify their cost functions to submit to the ICA next time.

The concept of finding the optimal price from the intersection of the aggregate GENCO's incremental cost curve and the aggregate DISTCO's decremental revenue curve in subsection 3.1.2.1 will be apparent from the following. From the optimal solution in the Table 4.10, GENCO 3 and DISTCO 3 are committed in stage 1. GENCOs 2, 3 and DISTCOs 1, 2 are committed in stage 2. All GENCOs

Table 4.10 Solution found by LR for subsection 4.1.2

Company	$t=1$	$t=2$	$t=3$	$t=4$
GENCO 1	-	-	500	-
GENCO 2	-	320	400	-
GENCO 3	170	200	200	200
DISTCO 1	170	350	600	150
DISTCO 2	-	170	350	50
DISTCO 3	-	-	150	-

Table 4.11 Optimal GENCO costs and DISTCO revenues for all stages for subsection 4.1.2

	GENCO 1	GENCO 2	GENCO 3	DISTCO 1	DISTCO 2	DISTCO 3
Cost or Revenue	6000.00	7016.00	5764.50	17180.00	7687.15	1956.25

Table 4.12 Price from EDC of each stage for subsection 4.1.2

Stage	1	2	3	4
Price from EDC	7.70	9.60	12.00	9.60

and DISTCOs are committed in stage 3. GENCO 3 and DISTCO 1, 2 are committed in stage 4. Fig 4.1, 4.2, 4.3, and 4.4 show the intersections of the aggregate committed GENCO incremental cost curves and the aggregate committed DISTCO decremental revenue curves in stages 1, 2, 3, and 4 respectively. The intersections taken from the curves give the same optimal prices as those shown in Table 4.12.

4.1.3 Applying LR to type 3 auctions

In subsection 4.1.3 and 4.1.4, the prices of the bids are in $\$/MWH$. The amounts of the bids and the loads are shown in MW, but actually they are implemented in p.u. of 100 MVA base. GENCOs 1, 2, and 3 are at buses 1, 2, and 3 respectively. The bids submitted by all the GENCOs are shown in Table 4.13 and the changes of loads at buses 4, 5, and 6 are shown in Table 4.14. From the load change data and the network data in the Appendix, F, which is defined in (3.75) is calculated and equal to 0.456 p.u. The optimal result is shown in Table 4.15. Optimal change of the total real power loss is calculated and is equal to 1.37 MW. The optimal revenue of sellers is \$470.25. The concept of finding the optimal λ from the value of λ of the aggregate λ curve of sellers at the value F of the horizontal axis in subsection 3.1.4.3 is shown in Fig. 4.5. The optimal λ is 12.452.

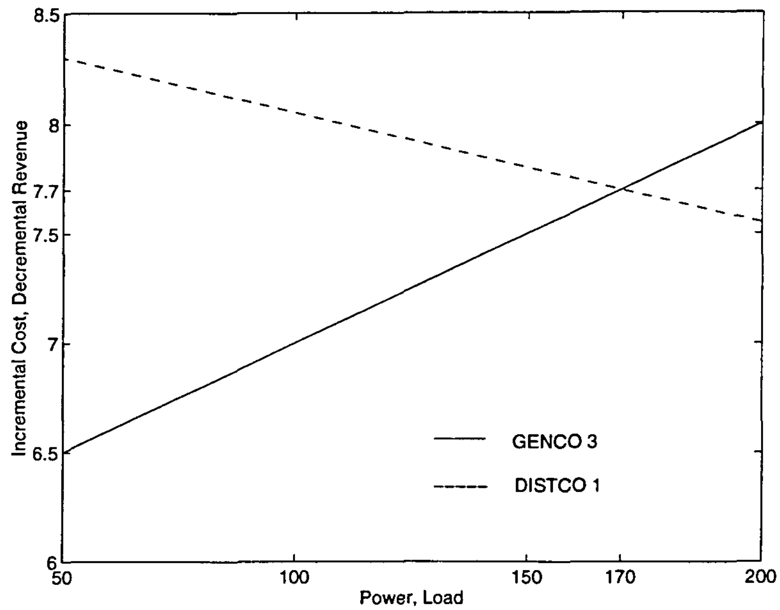

Figure 4.1 Intersection of aggregate incremental cost and aggregate decremental revenue at $t = 1$

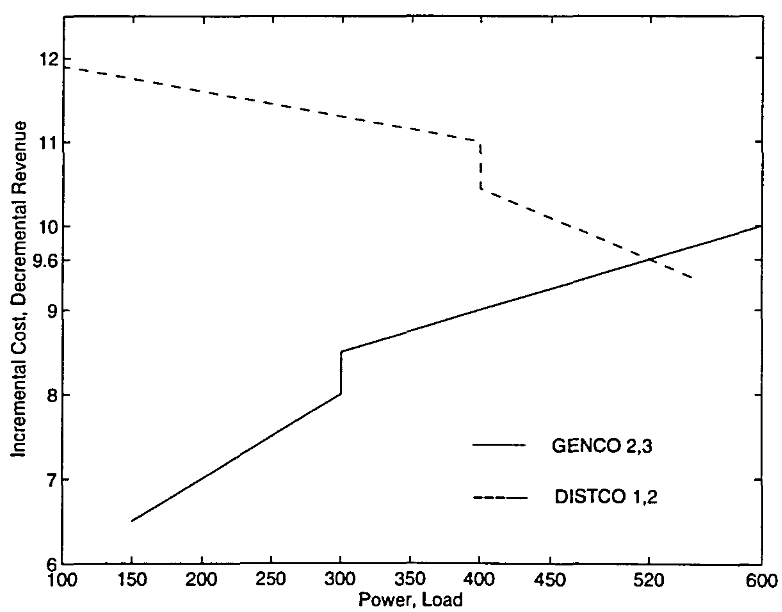

Figure 4.2 Intersection of aggregate incremental cost and aggregate decremental revenue at $t = 2$

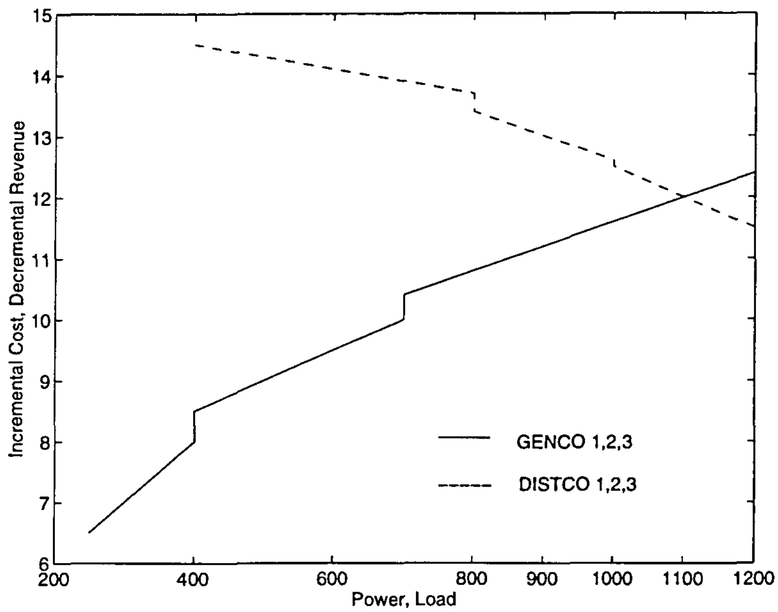

Figure 4.3 Intersection of aggregate incremental cost and aggregate decremental revenue at $t = 3$

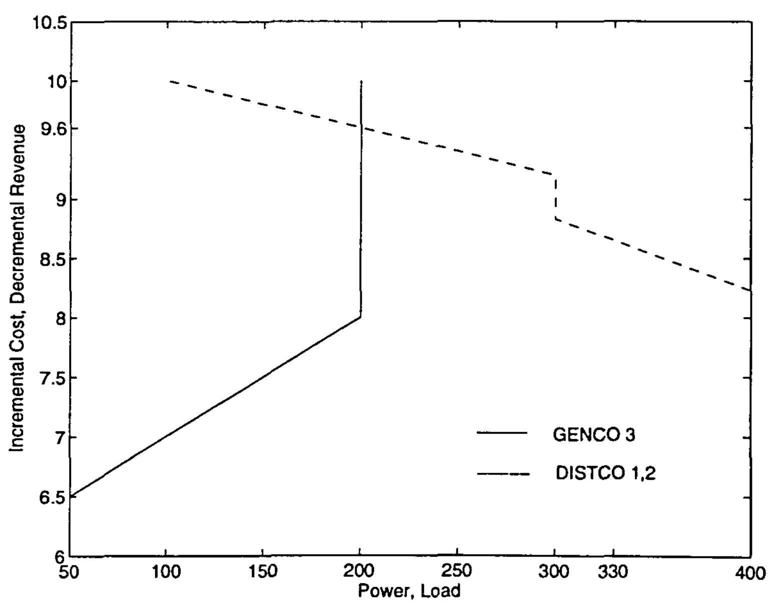

Figure 4.4 Intersection of aggregate incremental cost and aggregate decremental revenue at $t = 4$

Table 4.13 Bid data for subsection 4.1.3

Bid i	GENCO 1 $i=1$	GENCO 2 $i=2$	GENCO 3 $i=3$
Price, c_{si}	9.70	8.80	7.00
Amount, B_{si}	20.00	25.00	20.00

Table 4.14 Load change data for subsection 4.1.3

Bus j	4 $j=1$	5 $j=2$	6 $j=3$
Load change, ΔP_{bj}	25.00	10.00	20.00

Table 4.15 Accepted bids for subsection 4.1.3

Bid	GENCO 1	GENCO 2	GENCO 3
Price, c_{si}	9.70	8.80	7.00
Amount, ΔP_{si}	11.37	25.00	20.00

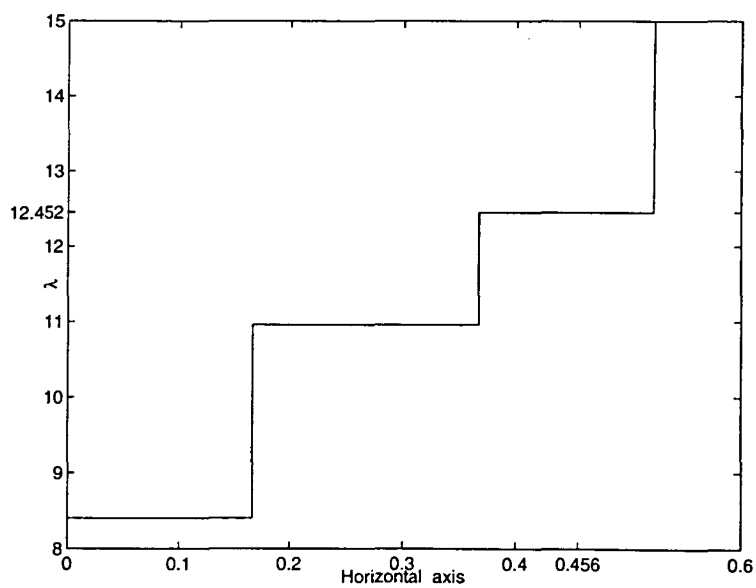

Figure 4.5 Optimal λ from the value of λ of the aggregate λ curve of sellers at the value F of the horizontal axis

From the result in Table 4.15, we can see that GENCOs 2 and 3 can sell all power offered but GENCO 1 can sell only 11.37 MW from the offered amount of 20 MW. The result is not only because GENCO 1 offers the highest bids and the total change of demand of the market, 55 MW, is less than the total change of supply of the market, 65 MW, it also depends on the network constraints. Sometimes even the GENCO which bids the lowest will have the contract limited to the network constraints. GENCO 1 might have to reduce its asking price in the next time period.

4.1.4 Applying LR to type 4 auctions

GENCOs 1, 2, and 3 are at buses 1, 2, and 3 respectively and DISTCO 1, 2, and 3 are at buses 4, 5, and 6 respectively. The bids submitted by all the GENCOs and DISTCOs are shown in Table 4.16. The optimal result is shown in Table 4.17. The change of the total real power loss is calculated and is equal to 0.52 MW. The optimal surplus is $135.06. The concept of finding the optimal λ from the intersection of the aggregate λ curve of sellers and the aggregate λ curve of buyers in subsection 3.1.4.4 is shown in Fig. 4.6. The optimal λ is 11.287.

Table 4.16 Bid data for subsection 4.1.4

Bids i, j	GENCO 1 $i = 1$	GENCO 2 $i = 2$	GENCO 3 $i = 3$	DISTCO 1 $j = 1$	DISTCO 2 $j = 2$	DISTCO 3 $j = 3$
Price, c_{si}, c_{bj}	9.70	8.80	7.00	12.00	10.50	9.50
Amount, B_{si}, B_{bj}	20.00	25.00	20.00	25.00	10.00	20.00

Table 4.17 Accepted bids for subsection 4.1.4

Bids i, j	GENCO 1 $i = 1$	GENCO 2 $i = 2$	GENCO 3 $i = 3$	DISTCO 1 $j = 1$	DISTCO 2 $j = 2$	DISTCO 3 $j = 3$
Price, c_{si}, c_{bj}	9.70	8.80	7.00	12.00	10.50	9.50
Amount ΔP_{si}, ΔP_{bj}	0.00	25.00	20.00	25.00	10.00	9.48

From the result in Table 4.17, we can see that GENCOs 2 and 3 can sell all power offered for sale and DISTCOs 1 and 2 can buy all power they bid on. DISTCO 3 can buy only part of the desired power and GENCO 1's bid is not accepted. The reason is not only because GENCO 1 offers the highest bid and DISTCO 3 offers the lowest bid, it also depends on the network constraints. Sometimes even the GENCO which bids the lowest or the DISTCO which offers the highest bid will have their contracts limited due to the network constraints.

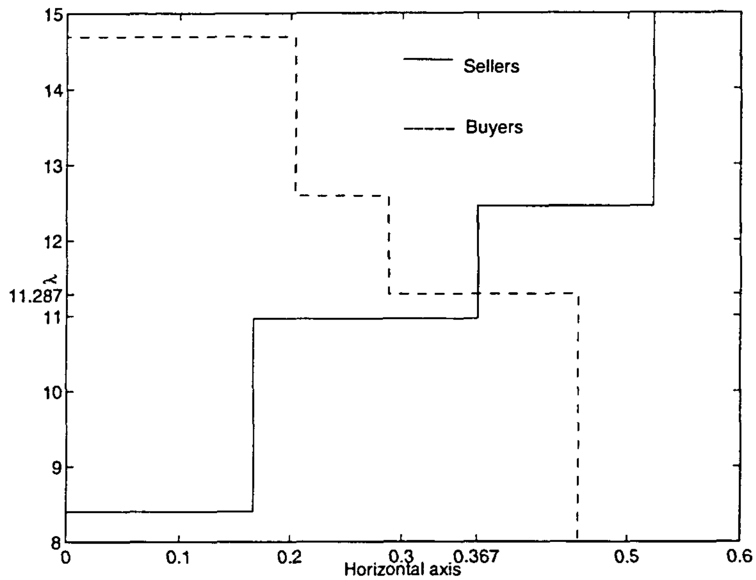

Figure 4.6 Optimal λ from the intersection of the aggregate λ curve of sellers and the aggregate λ curve of buyers

4.1.5 Implementation problems in applying LR to type 1 and 2 auctions

The auctions used to demonstrate the problems here are type 1 auctions. The system under investigation in this subsection is composed of four GENCOs (modified from Wood et. al. [34]) and these GENCOs are committed for four stages. Each GENCO has one generating unit.

The implementation problems can be separated into two main categories: Problems arising from identical units and problems arising from similar units.

4.1.5.1 Problems with identical units

There are two main problems when identical units exist. The first is that LR may find only suboptimal solutions. The second is that LR may be unable to find any feasible solutions. For this subsection start-up cost is not considered because it does not affect the solution found by LR.

4.1.5.1.1 Finding only sub-optimal solutions The generating unit data for this subsection is described in Table 4.18. Units 1, 2, and 3 here are the same as Units 1, 2, and 3 in subsection 4.1.1. Unit one is identical to unit four. Unit three is the least expensive unit, and units one and four are the most expensive units. System loads are shown in Table 4.19.

The solution found by LR is shown in Table 4.20. The solution found by LR is not the optimal solution. It is different from the optimal solution at the third stage, in which either unit 1 or 4 is selected to generate at 500 MW. This is evident from the total cost of the optimal solution, $20162.75, which is less expensive than that of the solution which LR found, $20412.75. The difference is more pronounced when the start-up costs are taken into account.

The problem occurs because LR uses DP to find the optimal states for subproblems, and for DP, identical or very similar units must have the same optimal states for the subproblems. This is why LR cannot find the optimal solution which selects either unit 1 or 4 at the third stage. This problem means that the solution found by LR may not be the least expensive nor the best for the whole system when identical or very similar units exist.

Table 4.18 Generating unit data for subsection 4.1.5.1.1

GENCO i	a_{ig}	b_{ig}	c_{ig}	P_{ig}^{min}	P_{ig}^{max}
GENCO 1	0.002	10	500	100	600
GENCO 2	0.0025	8	300	100	400
GENCO 3	0.005	6	100	50	200
GENCO 4	0.002	10	500	100	600

Table 4.19 Load data for subsection 4.1.5.1.1

Stage	1	2	3	4
Load	170	520	1100	330

Table 4.20 Solution LR found for subsection 4.1.5.1.1

Stage(t)	GENCO 1	GENCO 2	GENCO 3	GENCO 4
1	0	0	170	0
2	0	320	200	0
3	250	400	200	250
4	0	130	200	0

4.1.5.1.2 Not finding any feasible solutions For this subsection units 1 and 4 are still the identical units. The system load at the third stage is changed to be between the summation of P_{ig}^{min} of units 1, 2, 3, and that of units 1, 2, 3, 4. In addition, P_{ig}^{max} of units 2 and 3 are reduced so that the load at the third stage cannot be met by only selecting units 2 and 3. The purpose of changing data in this way is to force only either unit 1 or 4 to be selected at the third stage. The loads at other stages are reduced to accommodate the decreased total maximum capacity. The generating unit data is the same as in Table 4.18, except that P_{ig}^{max} of units 2 and 3 are changed to 150, and 80 respectively. The load data is shown in Table 4.21.

Three starting λ (λ_a, λ_b, and λ_e) have been used for running LR. After running 100 iterations for each starting λ, LR could not find any feasible solutions. The reason can be explained as follows. To cover the load at the third stage at the lowest cost, unit 2 and 3 must be selected. For unit 1 and 4, there are only two possible combinations of states; both units are either selected or not selected. The case in which both units are not selected cannot occur because the summation of P_{ig}^{max} of units 2 and 3 are less than 340. The case in which both units are selected cannot occur because the summation of P_{ig}^{min} of units 1, 2, 3 and 4 are larger than 340. When considering only the fourth stage, LR can find the solution which is not optimal and this is the case explained in subsection 4.1.5.1.1. The curves showing updated primal and dual objective functions of starting λ_b are shown in Fig. 4.7. Note that a big value of primal objective function is used for the stage having not enough committed generation.

The example studied in this subsection points out another disadvantage of using LR for auctions when identical and very similar units exist. Not only is the solution found by LR probably not the real optimal, but it is also sometimes difficult for LR to even find a feasible solution.

Table 4.21 Load data for subsection 4.1.5.1.2

Stage	1	2	3	4
Load	80	210	340	350

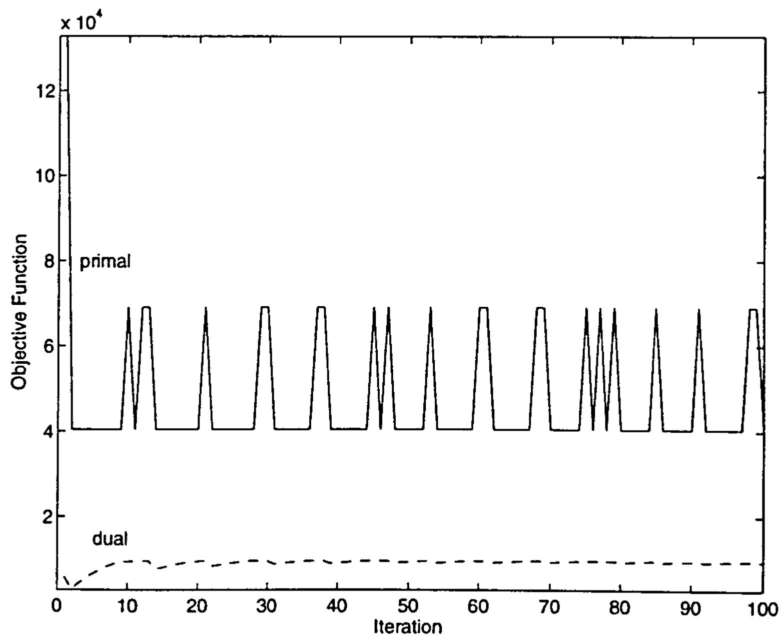

Figure 4.7 Updated primal and dual objective functions of subsection 4.1.5.1.2

The concluding remark of subsections 4.1.5.1.1 and 4.1.5.1.2 can be made with the economic interpretation of the LR iterations which will be described as follows: If an energy market is considered, the LR algorithm proposes a sequence of hourly prices (λ) to buy energy from GENCOS. GENCOS, each one independently, plan their output power in response to the price sequence, meeting their respective constraints. This results in a surplus of power in some hours and deficit of power in some other hours. The sequence of prices is then modified by the LR algorithm with the objective to balance demand. A reasonable procedure is to modify prices proportionally to their corresponding mismatches (subgradient). This procedure is repeated until convergence in prices is attained. These prices are in turn implemented. A reserve market working in a similar fashion as the energy market can also be implemented.

From the above description it directly follows that identical units will be jointly selected or not selected. However, it is not possible that some of them will be selected while the rest are not. This produces two problematic behaviors: First, it is possible to miss the minimizer, should it require that some of the identical units be selected and not the rest (subsection 4.1.3.1.1). Second, it is possible not to find any feasible solutions. This happens if the selection of all identical units in a given hour produces infeasibility in demand because the total minimum output power is larger than the demand; and the not selection of all the identical units in a given hour makes it impossible to supply the demand (subsection 4.1.3.1.2).

The rules to solve the problems with identical units may be constructed to make identical units not identical while *preserving fairness*; for instance, they can be penalized in a rotating and cycling fashion. However, the rules that can preserve *fairness for every unit* are very difficult to construct.

4.1.5.2 Problems with similar units having multiple optimal solutions

The data studied in this subsection is described in Table 4.22 and Table 4.23. Unit 1 is similar to unit 4. The start-up cost is added for units 1 and 4 which are peak units.

Table 4.22 Generating unit data for subsection 4.1.5.2

GENCO i	a_{ig}	b_{ig}	c_{ig}	P_{ig}^{min}	P_{ig}^{max}	$stup_{ig}$
GENCO 1	0.0020	10.00	500	100	600	3300.7
GENCO 2	0.0025	8.00	300	100	400	0
GENCO 3	0.0050	6.00	100	50	200	0
GENCO 4	0.0020	9.88	542	100	600	3324.7

Table 4.23 Load data for subsection 4.1.5.2

Stage	1	2	3	4
Load	170	520	1100	1000

Two approaches, using different starting λ and changing the order of the unit data as it is fed to the program (alternating between the two peak units, units 1 and 4), are tested with this subsection. LR is run for two unit data input orders, unit order 1 2 3 4 and 4 2 3 1, and for each unit data input order, five starting $\lambda(\lambda_a, \lambda_b, \lambda_c, \lambda_d$, and $\lambda_e)$ are used to run LR. LR is run for 100 iterations for each case. In 100 iterations LR may find the optimal solution more than once. The reason that LR is run for fixed number of iterations, 100, instead of running until the duality gap is satisfied, is to find out whether some different optimal solutions are found in the large number of iterations or not.

The result can be explained by the following. The unit data input order does not affect solution, i.e., unit order 1 2 3 4 and 4 2 3 1 give the exactly same solution. The optimal solutions found by LR in all different starting λ are the same. For some starting λ when LR found optimal solutions more than once, they are still the same. In conclusion, LR found only one optimal solution as shown in Table 4.24.

Actually there are two optimal solutions for this subsection. One is what LR found (shown in Table 4.24). The other is shown in Table 4.25. λ_c ,which corresponds to the optimal λ of the solution in Table 4.24, is used to test the hypothesis that if it is used as a starting λ, LR will find the other optimal solution in Table 4.25. This does not happen.

Various starting λ and two different unit data input orders have been used to obtain the results, yet only one optimal solution is found by LR. The optimal solution found is one, in which LR selects unit 4 at the third and fourth stages, but actually unit 1 could have been selected and would have provided the same total cost, \$30801.2. Thus, this is unfair to unit 1.

In our deregulated competitive environment, similar generating units will be prevalent. Therefore using LR as an auction method may be inequitable to some generating units. These units might not be selected by LR, even though these units can provide the same total cost as the units that LR selected.

Table 4.24 Optimal solution LR found for subsection 4.1.5.2

Stage(t)	GENCO 1	GENCO 2	GENCO 3	GENCO 4
1	0	0	170	0
2	0	320	200	0
3	0	400	200	500
4	0	400	200	400

Table 4.25 The alternative optimal solution for subsection 4.1.5.2

Stage(t)	GENCO 1	GENCO 2	GENCO 3	GENCO 4
1	0	0	170	0
2	0	320	200	0
3	500	400	200	0
4	400	400	200	0

4.1.6 Sensitivity analysis

In subsection 4.1.5.1.1 we saw that LR was unable to find the optimal solution when identical units exist. This section uses the generating unit data and load data of subsection 4.1.5.1.1 to do sensitivity analysis for each of these three parameters, c_{ig}, b_{ig}, and $stup_{ig}$ of unit 1 and 4. The only additional data is \$3000 of start-up cost for both of units 1 and 4. The procedure varies each of c_4, b_4, and $stup_4$ individually in the amount of -10% to 10% of the original value, in increments of 1%, the results are shown in Fig. 4.8, 4.9, and 4.10. The sensitivity analysis results are coupled with the subgradient updating procedure.

Fig. 4.8 represents the curves obtained by varying c_4. The curves obtained by varying b_4 are the same as those of varying $stup_4$ and they are shown commonly in Fig. 4.9. The curves of Fig. 4.8 and 4.9 are plotted between normalized cost and percentage of difference of parameters of units 4 and 1. The normalized cost is found by comparing the cost of LR's solution to the optimal cost. Actually three curves are shown in each figure according to each starting λ, but in Fig. 4.8 $\lambda_{\mathfrak{b}}$ and $\lambda_{\mathfrak{g}}$ give the same curve and in Fig. 4.9 all $\lambda_{\mathfrak{b}}$, $\lambda_{\mathfrak{f}}$, and $\lambda_{\mathfrak{g}}$ give the same curve. The optimal solution for 0% difference of each parameter is what is shown in Table 4.20 in which both units 1 and 4 are selected at the third stage. For the optimal solution of other percentage, unit 4 (not unit 1) shall be selected at the third stage for -10% to -1% difference because unit 4 is less expensive than unit 1 in this range, and unit 1 (not unit 4) shall be selected for 1% to 10% difference because unit 1 is less expensive in this range.

The curves of $\lambda_{\mathfrak{b}}$ and $\lambda_{\mathfrak{g}}$, in Fig. 4.8 show that the optimal solution can be found only if there is a difference in the parameters. If two or more units have similar values, then it is hard for the algorithm to select between the two. Fig. 4.9 demonstrates the same problem. It is especially important to examine the curve of $\lambda_{\mathfrak{f}}$ in Fig. 4.8 since a 4 % change is needed to distinguish between the units for a negative change. Note that the change is not symmetric with respect to the origin. Fig. 4.8 can be understood if the updating procedure is examined. It can be traced to discover that the optimal value of λ cannot be reached by the update algorithm from a value of $\lambda_{\mathfrak{f}}$. The problem exists primarily at the peak demand condition, λ^3. At this level of operation the optimal solution cannot be found. It might be interesting

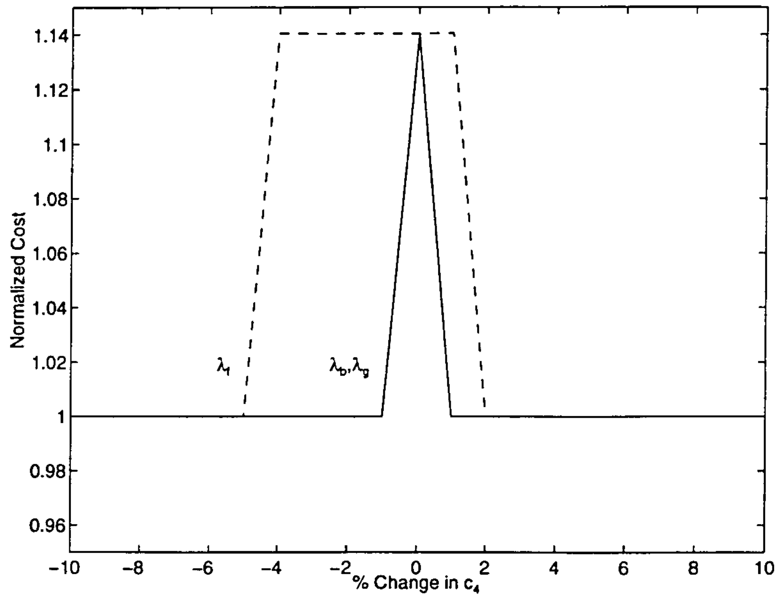

Figure 4.8 Normalized cost for varying c_4.

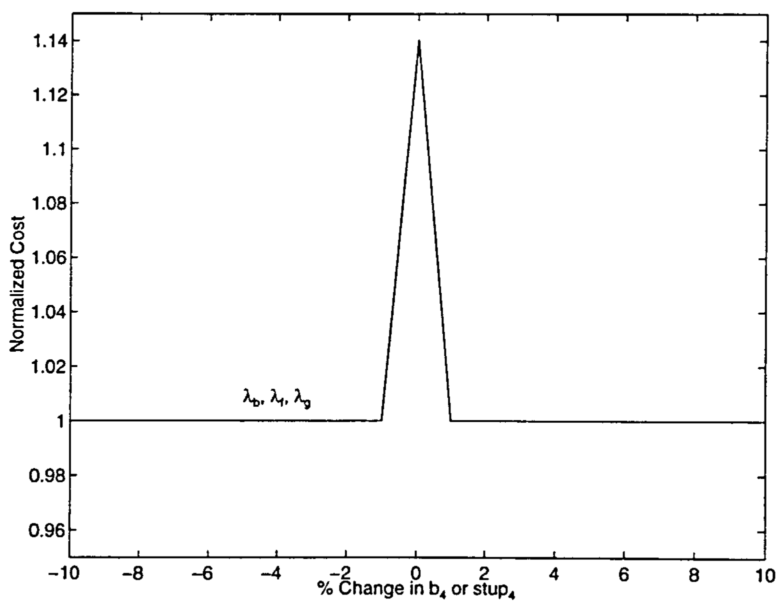

Figure 4.9 Normalized cost for varying b_4, or $stup_4$.

to ask what the range of optimal λ^3 is. Due to small system size, this question can be solved manually. The optimal λ^3 for each changed c_4 for which unit 1 is less expensive than unit 4 (positive difference percentage) is shown in (4.1) and the optimal λ^3 for each changed c_4 for which unit 4 is less expensive than unit 1 (negative difference percentage) is shown in (4.2).

$$17.0333 < \lambda^3 < 17.0333 + \frac{c_4 - c_1}{600} \tag{4.1}$$

$$17.0333 - \frac{c_1 - c_4}{600} < \lambda^3 < 17.0333 \tag{4.2}$$

From (4.1) and (4.2) the gap of optimal λ^3 is only $\frac{|c_4 - c_1|}{600}$, which is very small. For example, if $c_4 - c_1$ is 1%, $17.0333 < \lambda^3 < 17.0417$, and the gap is only 0.0084.

From the derived result it can be explained that the optimal range of λ^3 is very small when the percentage of difference of c_4 is small. And if vector λ is not updated properly with the system data and starting λ, LR cannot converge to the optimal solution. This is why starting λ can affect the solution. Although if $|c4 - c1| \neq 0$, (4.1) and (4.2) show that the optimal range of λ^3 exists. Practically it's difficult for LR to update λ^3 such that λ^3 is in this optimal range, especially when the range is small. This problem can probably be solved by using larger α and β in updating λ when $pdif^t$ is negative but this will make LR converge more slowly.

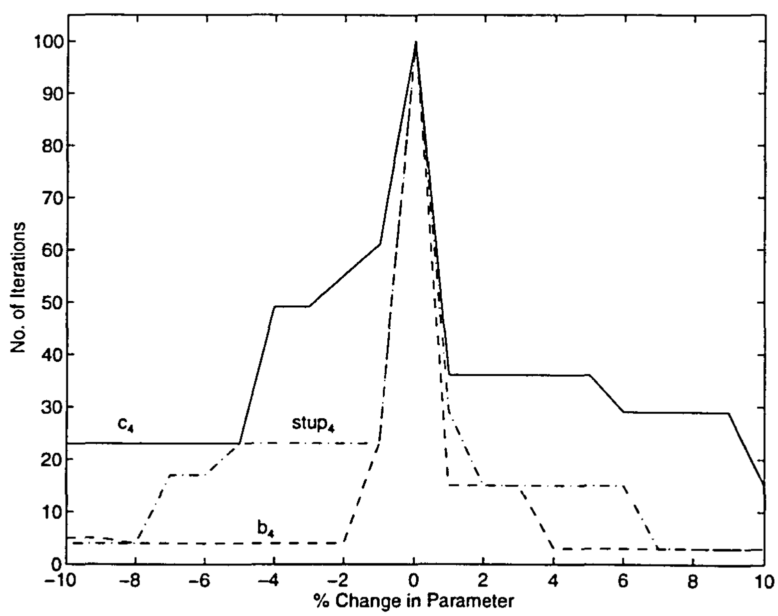

Figure 4.10 No. of iterations needed to find the optimal solution.

Fig. 4.10 illustrates the number of iterations needed to find the optimal solution versus percent difference of each parameter for starting λ_b. Three curves are shown according to each parameter. The curves for other starting λ are similar to Fig. 4.10 but they used more iterations than each curve in Fig. 4.10 due to effect of starting λ. The number of iterations is 100 at 0% difference and this means that LR cannot find the optimal solution. From Fig. 4.10, it can be seen that varying c_4 requires more iterations than varying b_4 or $stup_4$ and varying $stup_4$ requires a little bit more iterations than varying b_4. In addition, from Fig. 4.8 and Fig. 4.9 we see there is only one case of varying c_4 where LR cannot find the optimal solution. Because of this, it can be implied that for the same percent difference of each parameter between units 4 and 1, the order from the most difficult to the least difficult for LR convergence is varying c_4, $stup_4$, and b_4 respectively. This can be explained based on the result that either unit 1 or unit 4 is selected once at 500 MW at the third stage. Suppose the percent difference is p, then the difference between the cost of units 4 and 1 is $5 * p$ for varying c_4, $30 * p$ for varying $stup_4$, and $.1 * 500 * p = 50 * p$ for varying b_4. Varying b_4 produces the greatest difference in cost, while varying c_4 produces the smallest difference in cost. Increasing the differences in cost increase the ease with which LR can find the optimal solution. That is why for the same percentage of difference, varying c_4 produces the most difficulty for LR convergence and varying b_4 produces the least difficulty.

From the sensitivity analysis, it can be seen that when identical or similar units exist in the system, LR has difficulty in converging to the optimal solution. It is evident from (4.1) and (4.2) that the range of optimal λ is small, especially when $|(c_4 - c_1)|$ is small. The system studied here is very small. For real systems which are much bigger and more complex, the range of optimal λ is an interesting future research topic. If the range of optimal λ is smaller, LR will have more difficulty in converging to the optimal solution. Also, real systems are so big that the optimal λ range as derived in (4.1) and (4.2) cannot be derived to adjust α and β to update λ to be proper with the system. Moreover, The auction has a dynamic feature which changes every period and α and β which are valid with the auction in one period may be invalid with other periods.

4.2 Interior-Point Linear Programming (IPLP)

The two examples illustrated in this section that applies IPLP to type 3 and 4 auctions, are the same as those in subsections 4.1.3 and 4.1.4 of LR. The results of this section are also the same as those in subsections 4.1.3 and 4.1.4 of LR except the number of iterations for getting the optimal solutions. This issue is discussed in section 4.4. Note that the optimal values of the dual variables of the coupling constraints are the same as the values of the optimal λ in subsections 4.1.3 and 4.1.4, 12.452 and 11.287

respectively.

4.3 Upper-Bound Linear Programming (UBLP)

The two examples illustrated in this section that applies UBLP to type 3 and 4 auctions, are the same as those in subsections 4.1.3 and 4.1.4 of LR. The results of this section are also the same as those in subsections 4.1.3 and 4.1.4 of LR except the number of iterations for getting the optimal solutions. This issue is discussed in section 4.4. Note that the optimal values of the dual variables of the coupling constraints are the same as the values of the optimal λ in subsections 4.1.3 and 4.1.4, 12.452 and 11.287 respectively.

4.4 Comparison among LR, IPLP and UBLP

The same two illustrative examples of type 3 and 4 auctions are implemented using LR, IPLP and UBLP. These three methods give the same results. The number of iterations for getting the optimal solutions varies with the method used. The number of iterations are compared in Table 4.10.

Table 4.26 Comparison of number of iterations for getting the optimal solutions of the two illustrative examples

Method	Type 3 Auctions	Type 4 Auctions
LR	3	3
IPLP	9	10
UBLP	3	5

From Table 4.26, we can see that the UBLP method uses fewer iterations than the IPLP method. This is typical of the simplex method which uses fewer iterations than the interior-point method for the small dimension problems. Additionally, the algorithm of the simplex method used in this thesis is the upper-bound method. This further reduces number of iterations because implementation by UBLP helps reduce the number of constraints which in turn helps reduce the computational and the storage requirements. These reasons are why the UBLP uses fewer iterations in the illustrated examples. For the larger dimensional auction problems, comparison of the number of iterations of both methods would be an interesting research topic.

Of the methods compared, LR uses the fewest iterations to find the optimal solution. This is due to the configuration of type 3 and 4 auction problems which facilitates updating as explained in subsection 3.1.3.4. For the larger dimensional auction problems, such as when there are additional constraints or when there are more sellers and buyers, comparing of the number of iterations of these three methods would be an interesting research topic.

5 CONCLUSIONS

In the U.S. electric power has been supplied by vertically integrated monopolistic utilities for a long time. Presently the electric power industry in the U.S. is restructuring to be more competitive. The cost-based approach to developing electricity rates will be changed to a price-based approach and auctions are considered to be a promising pricing mechanism for the competitive market. There have been various types of auctions proposed for use in the electric power market. This thesis focuses on four types of auctions.

The purpose of this thesis is to show how to implement auctions using LR, IPLP, and UBLP and to describe the problems associated with using LR to implement type 1 and 2 auctions.

LR is used to implement type 1, 2, 3, and 4 auctions. For type 1 and 2 auctions, the formulation, the algorithm, and the computer program for implementing a type 2 auction are very similar to those of type 1 auctions. Therefore, the algorithm and the computer program of type 1 auctions can be used with type 2 auctions with only slight modification. The concepts of quadratic and concave revenue functions and finding the optimal price via the intersection of aggregate GENCOs' incremental cost curve and DISTCOs' decremental revenue curves are discussed for type 2 auctions. For type 3 and 4 auctions, because the configuration for type 4 auctions is not very complex, instead of continuously switching between solving the primal and dual problems, the procedure of using LR to implement the auctions is reduced to a simple algorithm; and this makes the updating procedure simple also.

Different types of auctions are implemented well and efficiently with different methods. This thesis implements four types of auctions with various methods and this gives a good understanding of choosing the suitable method to implement each of the four types of auctions. This also gives insight on which of these methods to apply with other types of auctions.

The problems of using LR to implement type 1 and 2 auctions are described with illustrative examples. The illustrative examples are tested on type 1 auctions. The problems studied in this thesis are divided into two categories, problems with identical units and problems with similar units. For identical units, LR will always select or deny all the identical units simultaneously no matter what the optimal solution is. This means that LR will probably be unable to find the optimal solution and sometimes will not even be able to find a feasible solution. For similar units, sometimes the optimal solution requires selection of only some of these units. Any subsets of similar units can be selected for the optimal solution. However, not all units may be selected as this would cause overgeneration. This is inequitable to the unchosen units which actually could provide an alternative optimal solution. The problems shown give a good indication for the auctioneer to modify the current method or to find a new method to implement type 1 and 2 auctions. The optimal solution should always be found and the optimal solution found should be fair to every GENCO.

IPLP and UBLP are used to implement type 3 and 4 auctions. IPLP is efficient for large-scale linear programs except that IPLP cannot find the exact optimal extreme point. Sensitivity analysis cannot be performed in IPLP since it is at an interior-point without being computationally expensive. This thesis develops an algorithm such that IPLP can find the exact optimal extreme point and then sensitivity analysis can be performed with inexpensive computational requirements. This can save significant computational cost in implementing large-scale linear programs, including auctions.

The algorithm developed checks to see if the duality gap and primal feasibility are satisfied and ensures that the number of components of the estimate of reduced cost coefficient vector, $\mathbf{z}$ (defined in 3.90) which are very close to zero is equal to the number of constraints. If these conditions are satisfied, the estimated optimal basic variables are the variables having satisfied values of $\mathbf{z}$, i.e. $\mathbf{z_i}$ which are very close to zero. Those estimated optimal basic variables can be verified with the KKT conditions for optimality.

Finally, the results of the same two illustrative examples of type 3 and 4 auctions tested on LR, IPLP and UBLP are compared. The results show that all the three methods yield the same results but require different number of iterations. Of the methods compared, LR requires the fewest iterations because of the configuration of type 3 and 4 problems which makes the updating procedure simple. IPLP requires the largest iterations.

APPENDIX SIX BUS SYSTEM

The six-bus system used for the examples in subsections 4.1.3 and 4.1.4, sections 4.2 and 4.3 of Chapter 4 is based on Wood et. al. [34]. The six-bus network is shown in Fig. A.1. The line data is shown in Table A.1. The original voltage, the real and reactive powers of generator and the load at each bus is shown in Table A.2. Some data is shown in per unit with 100 base MVA and 230 base kV.

Table A.1 Line data of six-bus network

From Bus	To Bus	Line Resistance (pu)	Line Reactance (pu)	Half of Line Charging (pu)
1	2	0.100	0.200	0.020
1	4	0.050	0.200	0.020
1	5	0.080	0.300	0.030
2	3	0.050	0.250	0.030
2	4	0.050	0.100	0.010
2	5	0.100	0.300	0.020
2	6	0.070	0.200	0.025
3	5	0.120	0.260	0.025
3	6	0.020	0.100	0.010
4	5	0.200	0.400	0.040
5	6	0.100	0.300	0.030

Table A.2 Bus data of six-bus network

Bus No.	Voltage Magnitude (V)	Voltage Angle (degree)	Generation Real Power (MW)	Generation Reactive Power (MW)	Load Real Power (MW)	Load Reactive Power (MW)
1	1.0500	0.00	112.62	34.79	-	-
2	1.0500	-2.53	140.00	75.07	-	-
3	1.0700	-5.15	60.00	112.34	-	-
4	0.9754	-4.68	-	-	100.00	70.00
5	0.9677	-6.58	-	-	100.00	70.00
6	0.9930	-7.27	-	-	100.00	70.00

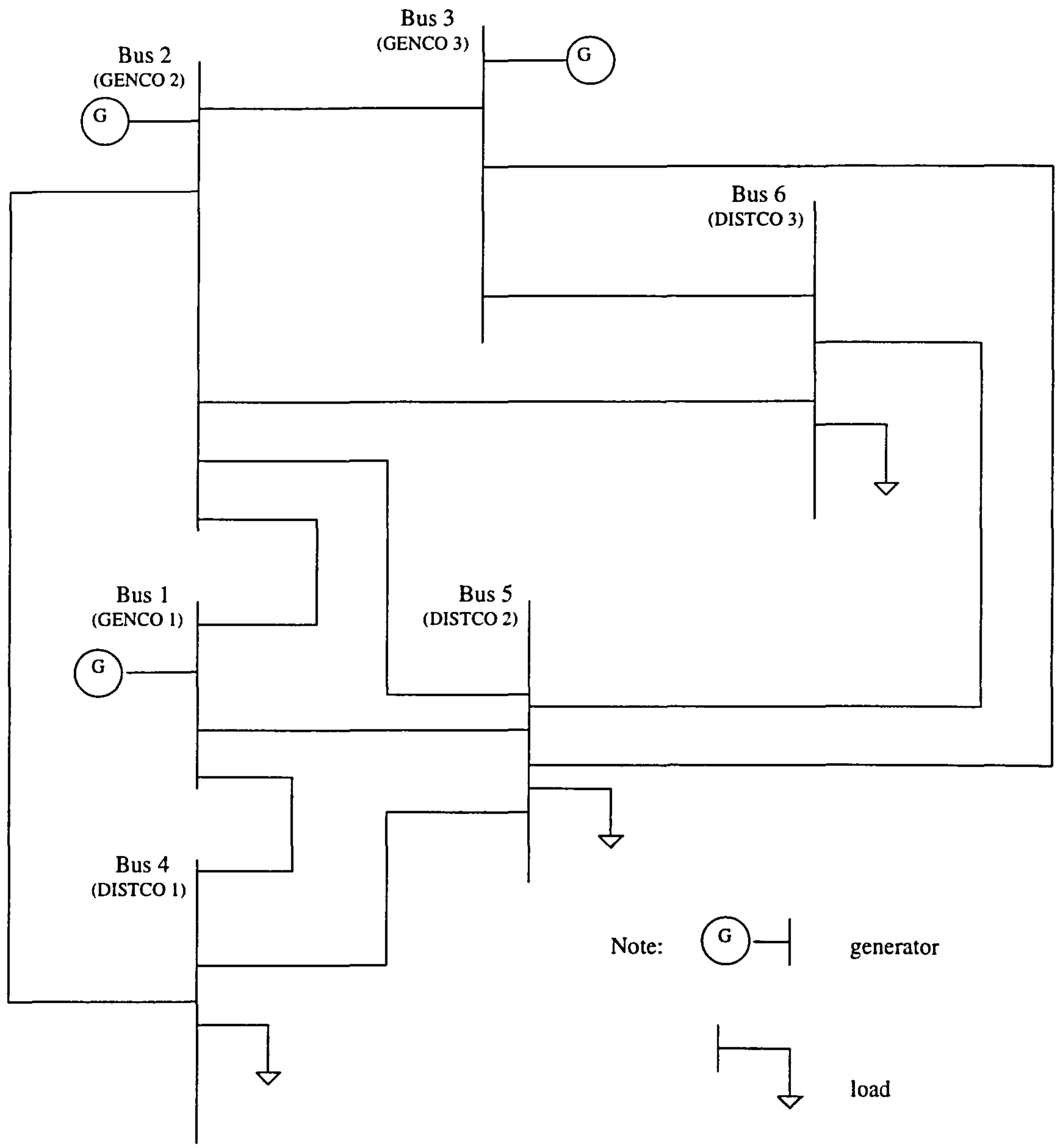

Figure A.1 Six-bus network

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